Name: Tuesday section time:
Student ID:

Math 20F - Linear Algebra - Winter 2003

Quiz #4 — February 11

(Do not discuss the quiz with students who haven't taken it yet – until 8:00pm.)

$$\text{Let } \mathbf{v}_1 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \ \mathbf{v}_2 = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}, \text{ and } \mathbf{v}_3 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}.$$

Answer the following questions. (As usual, be sure to justifiy your answers).

1. Are $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ linearly independent?
2. If not, show explicitly an example of how they are linearly dependent.
3. What is the dimension of $Span(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$?

ANSWER: If they are linearly dependent, there is a non-trivial solution to $\begin{pmatrix} 2 & 2 & -1 \\ 1 & 0 & -1 \\ -1 & 2 & 2 \end{pmatrix} \mathbf{x} = \mathbf{0}$. *We search for a non-trivial solution using row reduction:*

$$\begin{pmatrix} 2 & 2 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ -1 & 2 & 2 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & -1 & 0 \\ 2 & 2 & -1 & 0 \\ -1 & 2 & 2 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Thus, there are (infinitely) many non-trivial solutions, for example $x_3 = 2$, $x_2 = -1$, *and* $x_1 = 2$. *Thus,* $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ *are not linearly independent. An example of a linear dependence is:*

$$2\mathbf{v}_1 + (-1)\mathbf{v}_2 + 2\mathbf{v}_3 = \mathbf{0}.$$

The dimension of $Span(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ *is two. We know this because: (a) the dimension cannot be three because of the linear dependence, and (b) the dimension is not equal to one, since the three vectors are not multiples of a single vector.*