Math 20F - Linear Algebra - Winter 2003

Quiz #$3\frac{1}{2}$ — February 4

Answers

1. Let $S = \left\{ \binom{x}{y} \in \mathbb{R}^2 : x^2 \geq y \right\}$. Is S a subspace of $\mathbb{R}^2$?. Prove your answer.

   Don't be frightened by the word "Prove". It means the same as "Justify" or "Show your evidence". For this particular problem, showing a counter-example to a closure property is enough.

   ANSWER: We show that S is not closed under vector addition. For example, $\binom{1}{1}$ and $\binom{-1}{1}$ are in S. However

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \notin S.$$

2. Let $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$. Is $\{\mathbf{v}_1, \mathbf{v}_2\}$ a spanning set for $\mathbb{R}^3$. If not, give an example of a $\mathbf{x} \in \mathbb{R}$ which is not in $span(\mathbf{v}_1, \mathbf{v}_2)$.

   ANSWER: We need to determine whether there is a vector $(a\ b\ c)^T$ so that there is no solution to $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 = (a\ b\ c)^T$. We set it up a matrix equation

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

   and solve with row operations:

$$\left(\begin{array}{cc|c} 1 & 4 & a \\ 2 & 5 & b \\ 3 & 6 & c \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 4 & a \\ 0 & -3 & b-2a \\ 0 & -6 & c-3a \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 4 & a \\ 0 & -3 & b-2a \\ 0 & 0 & c-2b+a \end{array}\right)$$

   Thus, if $c - 2b + a \neq 0$, there is no solution and $(a\ b\ c)^T$ is not in $span(\mathbf{v}_1, \mathbf{v}_2)$. For example,

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \notin span(\mathbf{v}_1, \mathbf{v}_2).$$

   Thus, $\{\mathbf{v}_1, \mathbf{v}_2\}$ is not a spanning set for $\mathbb{R}$.