Math 20F - Study Outline: Basic Skills

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Now updated with material for the whole course.

This is a list of basic "skills" you should master for Math 20F. I have tried to make the list complete up through material for midterm #2, but of course you are also responsible for items that were inadvertantly omitted. There is a list of lecture topics on the course web page that you may also use for review purposes.

In addition to these skills, you are expected to **know definitions and theorems** and how to apply the definitions and theorems appropriately. You are responsible for material from the textbook, material in the course handouts, the material covered in class, and to a lesser extent the material in the Matlab assignments.

Chapter 1. :

- Convert a system of linear equations to matrix form, and vice-versa.
- Convert a matrix to reduced for echelon form or to RREF.
- Solve an (R)REF system by back substitution.
- Determine the number of solutions to a system of equations.
- Perform row operations.
- Perform matrix operations (addition, multiplication, scalar multiplication, etc.)
- Compute A^T.
- Determine if a matrix is singular.
- Compute A^{-1} if it exists.
- Work with elementary matrices and know their correspondence to elementary row operations.
- Put a matrix in LU form (if it has an LU form)
- Work with partitioned (i.e., blocked) matrices.

Chapter 2. :

- Calculate a determinant using cofactors.
- Calculate the determinant of a matrix using row operations.
- Calculate the determinant of a 2×2 matrix.
- Know the effect of row and column operations on the determinant.
- (We skipped Cramar's rule, and you are not responsible for knowing it.)

Chapter 3. :

- Determine if a subset of a vector space is a subspace. Know the closure conditions for a subspace.
- Know how to use vector space properties. (You do not need to memorise the list of axioms for a vector space.)
- Work with the vector spaces $R^{m\times n}$, P_n, $C[a,b]$, $C^n[a,b]$.
- Find the null space of a matrix.
- Know about linear combinations. Determine if a given vector in the span of a set of vectors.
- Find the dimension of the null space of a matrix, i.e. the nullity of the matrix.
- Determine if a given set of vectors is a spanning set for $\mathbb{R}^n$.
- Determine if a given set of vectors are linearly independent.
- Determine if a given set of vectors is a basis for $\mathbb{R}^n$.
- Given a set of vectors, find a linearly independent subset.
- (We skipped the Wronksian, and you are not responsible for knowing it.)
- Determine the dimension of a subspace.

- Find a basis for a subspace.
- Perform a change of basis.
- Find the matrix that performs a change of basis.
- Calculate the rank of a matrix.
- Find the row space, column space, null space and $N(A)$ of a matrix. Determine the dimensions of these spaces.

Chapter 4. :

- Represent a linear operator by a matrix.
- Find the matrix representation of a rotation.
- Express dot product and cross product with a matrix representation.
- Determine if a given transformation is linear.
- Find the image and kernel of a transformation. (Kernel is the same as nullspace and image is the same as range.)
- (We skipped homogeneous coordinates and you do not need to know them.)
- (We have skipped, at least for now, similarity in section 4.3.)

Chapter 5. :

- Compute scalar products.
- Find the magnitude of a vector.
- Find the angle between two vectors.
- Find the scalar and vector projection of a vector onto another vector.
- Find the orthogonal complement of a subspace.
- Know the complementary properties of $R(A^T)$ and $N(A)$, and of $R(A)$ and $N(A^T)$.
- Solve least squares problems.
- Find the best linear fit to data.
- Find the best quadratic fit to data.
- Find the projection of a vector $\mathbf{b}$ onto a subspace given as the span of arbitrary vectors.
- Express the projection as a matrix.
- Find the projection of a vector $\mathbf{b}$ onto a subspace given as the span of orthogonal vectors. (Also, onto a subspace given as the span of orthonormal vectors.)
- Recognize and use inner product notation.
- (For now at least, we skipped the use of function spaces as inner product spaces in section 5.4.)
- Determine if a set of vectors is orthogonal.
- Determine if a set of vectors is orthonormal.
- Determine if a matrix is orthogonal.
- Use the matrix method to find the projection of $\mathbf{b}$ onto a subspace given as a span of orthogonal vectors.
- (We skip, for now at least, permutation matrices and orthogonality in vector spaces of functions on page 275 and pages 279-285.)
- Solve least squares problems with orthogonal matrix.
- Gram-Schmidt method or modified Gram-Schmidt method to find an orthonormal basis of a subspace.

Chapter 6. :

- Find the eigenvalues of a matrix.
- Find the eigenvectors or eigenspace corresponding to an eigenvector.
- Compute the trace of matrix.
- Compute the product and sum of the eigenvalues of a matrix.
- Find complex eigenvalues.
- Determine if a matrix is diagonalizable. If so, find a diagonalization.
- Determine if a matrix is defective.
- Use similarity to convert matrices to different representation.