Name: Thursday section time:
Student ID:

Math 20F - Linear Algebra - Spring 2003

Quiz #6 — May 29

(Do not discuss the quiz with students who haven't taken it yet – until 8:00pm.)
You must show your work in order to get credit for a problem. **Label your answers clearly.**

1. Let $\mathbf{u}_1 = (1,1,1,1)^T$, $\mathbf{u}_2 = (1,-1,1,-1)^T$ and $\mathbf{u}_3 = (1,0,-1,0)^T$. Are these three vectors orthogonal? Are they orthonormal?

ANSWER: They are orthogonal, but not orthonormal.

2. Let $\mathbf{x} = (1,2,3,4)^T$. Find the projection of $\mathbf{x}$ onto $Span(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$.

ANSWER: We start by converting $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ *into an orthonormal basis, by letting*

$$\begin{aligned}
\mathbf{v}_1 &= \mathbf{u}_1/||\mathbf{u}_1|| = (1/2,\ 1/2,\ 1/2,\ 1/2)^T \\
\mathbf{v}_2 &= \mathbf{u}_2/||\mathbf{u}_2|| = (1/2,\ -1/2,\ 1/2,\ -1/2)^T \\
\mathbf{v}_3 &= \mathbf{u}_3/||\mathbf{u}_3|| = (1/\sqrt{2},\ 0,\ -1/\sqrt{2},\ 0)^T.
\end{aligned}$$

Then, find the projection $\mathbf{p}$ as

$$\begin{aligned}
\mathbf{p} &= \langle \mathbf{v}_1, \mathbf{x} \rangle \mathbf{v}_1 + \langle \mathbf{v}_2, \mathbf{x} \rangle \mathbf{v}_2 + \langle \mathbf{v}_3, \mathbf{x} \rangle \mathbf{v}_3 \\
&= 5\mathbf{v}_1 - \mathbf{v}_2 - \sqrt{2}\mathbf{v}_3 \\
&= (\tfrac{5}{2},\ \tfrac{5}{2},\ \tfrac{5}{2},\ \tfrac{5}{2})^T - (\tfrac{1}{2},\ -\tfrac{1}{2},\ \tfrac{1}{2},\ -\tfrac{1}{2})^T - (1,\ 0,\ -1,\ 0)^T \\
&= (1,\ 3,\ 3,\ 3)^T.
\end{aligned}$$