Name: Thursday section time:
Student ID:

Math 20F - Linear Algebra - Spring 2003

Answers for Self-assessment Quiz #6.5 — June 5

1. Let A be the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 5 \end{pmatrix}$.

Answer the following questions: What are the eigenvalues of A? For each eigenvalue, what is the the dimension of its eigenspace? Is A diagonalizable? Is A defective?

ANSWER: These questions can be answered without having to do any hard calculations. Since A is triangular, its eigenvalues are the diagonal entries, $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 5$. Since the eigenvalues are distinct, each eigenspace has dimension 1, A is diagonalizable and A is not defective.

2. Now let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ -3 & 0 & 5 \end{pmatrix}$. Find all of A's eigenvalues and eigenvectors.

ANSWER: To find the eigenvalues, we have $A - \lambda I = \begin{pmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 0 \\ -3 & 0 & 5-\lambda \end{pmatrix}$

Using the cofactor method (across the middle row is easiest in this case), we get $\det(A - \lambda I) = (2-\lambda)^2(4-\lambda)$. The first eigenvariable is $\lambda_1 = 2$: its associated eigenspace is dimension 1 and $(1,0,1)^T$ is an eigenvector for the eigenvalue λ_1. The second eigenvariable is $\lambda_2 = 4$ and its eigenspace also has dimension 1 and $(1,0,3)^T$ is a eigenvector for the eigenvalue λ_2.

Test-taking suggestion: you can check the correctness of the eigenvalues and eigenvectors by multiplying with A!