Name: Thursday section time:
Student ID:

Math 20F - Linear Algebra - Spring 2003

Quiz #5 — May 15

(Do not discuss the quiz with students who haven't taken it yet – until 8:00pm.)
You must show your work in order to get credit for a problem. **Label your answers clearly.**

1. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by the following procedure. To calculate $f(\mathbf{x})$, first rotate $\mathbf{x}$ counter-clockwise around the origin through an angle of 45° and, second, orthogonally project the resulting vector onto the x-axis. What matrix A represents the function f?

ANSWER: $A = \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & 0 \end{pmatrix}$

2 Let $\mathbf{u}_1 = (-1, 0)^T$ and $\mathbf{u}_2 = (1, 2)^T$. The vectors $\mathbf{u}_1, \mathbf{u}_2$ form a basis for $\mathbb{R}^2$ (you do **not** need to prove they form a basis). Define the linear function $g : \mathbb{R}^2 \to \mathbb{R}^2$ so that

$$g(\mathbf{x}) = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},$$

where b_1 and b_2 are the coordinates of $\mathbf{x}$ with respect to the basis $\mathbf{u}_1, \mathbf{u}_2$. Give the matrix B that represents g.

ANSWER: $B = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 1/2 \\ 0 & 1/2 \end{pmatrix}$