Name: Thursday section time:
Student ID:

Math 20F - Linear Algebra - Spring 2003

Answers to Self-assessment Quiz #5.5 — May 22

You must show your work in order to get credit for a problem. **Label your answers clearly.**

1. Let $\mathbf{u}_1 = (1,\ 1,\ 1,\ 0)^T$ and $\mathbf{u}_2 = (0,\ 1,\ 1,\ 1)^T$. Let $U = Span(\mathbf{u}_1, \mathbf{u}_2)$. Find a basis for $U^\perp$.

ANSWER: Form the matrix $A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$. *The nullspace of the matrix is equal to* $U^\perp$, *since* $R(A^T) = U$. *After finding also solutions to* $A\mathbf{x} = \mathbf{0}$ *(details of work omitted), we have*

$$U^\perp = Span((1,\ -1,\ 0,\ 1)^T, (0,\ 1,\ -1,\ 0)^T).$$

2 Find the linear function that best approximates data

x	-2	0	1	2
y	2	0	1	2

in the least squares sense.

ANSWER: Form the matrix $A = \begin{pmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}$. *Solve* $A\mathbf{x} = \mathbf{b}$, *where* $\mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ 1 \\ 2 \end{pmatrix}$. *For this, compute the normal equation* $(A^TA)\mathbf{x} = A^T\mathbf{b}$, *which is*

$$\begin{pmatrix} 4 & 1 \\ 1 & 9 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}.$$

This system has the solution $x_1 = 44/35$ *and* $x_2 = -1/35$. *Thus, the answer is* $(-1/35)x + 44/35$ *— this is the linear fuction that best approximates the data in the least squares sense.*