Name: Thursday section time:
Student ID:

Math 20F - Linear Algebra - Spring 2003

Quiz #3 — May1

(Do not discuss the quiz with students who haven't taken it yet – until 8:00pm.)
You must show your work in order to get credit for a problem. **Label your answers clearly.**

Let $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$, $\mathbf{u}_3 = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$, and $\mathbf{u}_4 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$,

1. Is $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ a spanning set for $\mathbb{R}^3$? Explain why or why not.

2. Is $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ a basis for $\mathbb{R}^3$? Explain why or why not.

3. Is $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$ a spanning set for $\mathbb{R}^3$? Explain why or why not.

4. Is $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$ a basis for $\mathbb{R}^3$? Explain why or why not.

ANSWERS:

1. *Yes, it is a spanning set. To prove this, you may do one of the following: (a) Form the matrix $A = (\mathbf{u}_1\ \mathbf{u}_2\ \mathbf{u}_3)$, put A in row echelon form and note that the row echelon form does not have any row of all zeros. Or, (b) Form A as in (a) and show that A is non-singular. To carry out either (a) or (b) use row operations:*

$$\begin{pmatrix} 1 & 1 & 4 \\ 0 & 2 & 2 \\ 1 & 4 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 4 \\ 0 & 2 & 2 \\ 0 & 3 & -3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 4 \\ 0 & 1 & 1 \\ 0 & 3 & -3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & -6 \end{pmatrix}.$$

2. *Yes, it is a basis. We know this from the theorem that states that, in a vector space of dimension 3, any spanning set of size 3 is a basis. (This theorem holds more generally for any value of n in place of 3.)*

3. *Yes, it is a spanning set. We know this because it contains the spanning set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ as a subset.*

4. *No it is not a basis. Any basis for $\mathbb{R}^3$ must contain exactly three vectors.*