Name: Thursday section time:
Student ID:

Math 20F - Linear Algebra - Spring 2003

Self-Assessment Quiz #2.5 — April 25

1. Find the determinant of the following matrix:

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 2 & 2 \\ 2 & 3 & 6 & 4 \\ -1 & 0 & 2 & 2 \end{pmatrix}.$$

ANSWER:

$$\begin{vmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 2 & 2 \\ 2 & 3 & 6 & 4 \\ -1 & 0 & 2 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & -2 \\ 0 & -1 & 4 & 0 \\ 0 & 2 & 3 & 4 \end{vmatrix} = (-1)\begin{vmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & 3 & 4 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 0 & -2 \end{vmatrix}$$

$$= (-1)\begin{vmatrix} 1 & 2 & 1 & 2 \\ 0 & -1 & 4 & 0 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & -2 \end{vmatrix} = (-1)(-1)\begin{vmatrix} 1 & 2 & 1 & 2 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 11 & 4 \\ 0 & 0 & 0 & -2 \end{vmatrix} = 22.$$

2. Let $\mathbf{v} = (-3,\, 3,\, 1,\, 1)^T$, $\mathbf{u}_1 = (1,\, 4,\, 3,\, 2)^T$, $\mathbf{u}_2 = (1,\, -2,\, -1,\, 0)^T$, and $\mathbf{u}_3 = (1,\, 1,\, 1,\, 1,\,)^T$. Is $\mathbf{v} \in \mathit{Span}(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$? Prove your answer. If "Yes", express $\mathbf{v}$ a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$.

ANSWER: Solve the equation $A\mathbf{x} = \mathbf{v}$, where $A = (\mathbf{u}_1\ \mathbf{u}_2\ \mathbf{u}_3)$:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 4 & -2 & 1 & 3 \\ 3 & -1 & 1 & 1 \\ 2 & 0 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -6 & -3 & 15 \\ 0 & -4 & -2 & 10 \\ 0 & -2 & -1 & 7 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -6 & -3 & 15 \\ 0 & -4 & -2 & 10 \\ 0 & 0 & 0 & 2 \end{array}\right).$$

(The last step subtracts $\frac{1}{2}$ times the third from the fourth row.) Then, there is no solution to $A\mathbf{x} = \mathbf{v}$, so $\mathbf{v} \notin \mathit{Span}(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$.

BONUS ANSWER: I meant to ask the question with the vector $\mathbf{w} = (-3,\, 3,\, 1,\, -1)^T$ instead of $\mathbf{v}$. In this case the answer is obtained by solving the equation $A\mathbf{x} = \mathbf{w}$:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 4 & -2 & 1 & 3 \\ 3 & -1 & 1 & 1 \\ 2 & 0 & 1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -6 & -3 & 15 \\ 0 & -4 & -2 & 10 \\ 0 & -2 & -1 & 5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -2 & -1 & 5 \\ 0 & -4 & -2 & 10 \\ 0 & -6 & -3 & 15 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -2 & -1 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

By inspection, $A\mathbf{x} = \mathbf{w}$ has a solution, so $\mathbf{x}$ is in $\mathit{Span}(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$. There are in fact infinitely many solutions, each of can be used to express $\mathbf{w}$ as a linear combination of $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$. For example, $\mathbf{x} = (-1)\mathbf{u}_1 + (-3)\mathbf{u}_2 + \mathbf{u}_3$.