Math 20F - Homework 3 - Selected answers

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Section 2.2, Problem 3. In part (d), the determinant is equal to 2.

Section 2.2, Problem 12. Hint: Use the fact that $\det(AB) = \det(A)\det(B)$.

Section 2.2, Problem 13. If $AB = I$, then $\det(AB) = \det(A)\det(B) = 1$. In particular, $\det(A) \neq 0$, so A is nonsingular, so A^{-1} exists. Then, we have $A^{-1}(AB) = A^{-1}I = A^{-1}$, and also, $A^{-1}(AB) = (A^{-1}A)B = IB = B$. Hence, $B = A^{-1}$ and therefore $BA = I$.

The significance is that to check whether $B = A^{-1}$, it suffices to check that either $AB = I$ or $BA = I$ (instead of having to check both these.

Section 3.1, Problem 13. This is not a vector space. The following axioms fail: A3, A4, A5, A6. (To answer the problem, you only need to give *one* example of an axiom that fails, but you should show explicitly how it fails.)

Section 3.2, Problem 11. $\mathbf{y}$ in is $Span(\mathbf{x}_1, \mathbf{x}_2)$, but $\mathbf{x}$ is not.