

Start Time: Your name: Answer Key
 Stop Time: Integrity signature:

Suggested time limit 15 minutes, not counting download and upload. Please add explanation if over 20 minutes.

Definition: Recall that for v and w strings, $v \circ w = vw$ is the concatenation of v and w . Let the concatenation $R \circ S$ of two sets R and S of strings in Σ^* be the set of strings

$$R \circ S = \{v \circ w : v \in R \text{ and } w \in S\}.$$

For example, $\{00, 111\} \circ \{0, 1\} = \{000, 001, 1110, 1111\}$.

1. Let $\Sigma^* = \{0, 1\}$. Suppose R and S are subsets of Σ^* (i.e., they are unary relations.) Further suppose R and S are computably enumerable. Prove that $R \circ S$ is computably enumerable either by giving an algorithm that enumerates $R \circ S$ or by giving an algorithm that semidecides $R \circ S$.

Assumption: M enumerates R ,
 and N enumerates S

Algorithm

For $i = 1, 2, 3, \dots$

Run M for i steps, let R_i be the set of outputs obtained within i steps.

Run N for i steps. Let S_i be the set of outputs obtained within i steps

For each $u \in R_i$,
 For each $v \in S_i$,

Output $u \circ v$ (and continue running)

End-for

End-for

End-for

The above algorithm enumerates $R \circ S$. Hence $R \circ S$ is computably enumerable.