

Start Time: Your name: *Answer Key*  
 Stop Time: Integrity signature:

Suggested time limit 15 minutes, not counting download and upload. Please add explanation if over 20 minutes.

**Definition:** Recall that for  $v$  and  $w$  strings,  $v \circ w = vw$  is the concatenation of  $v$  and  $w$ . Let the concatenation  $R \circ S$  of two sets  $R$  and  $S$  of strings in  $\Sigma^*$  be the set of strings

$$R \circ S = \{v \circ w : v \in R \text{ and } w \in S\}.$$

1. Let  $\Sigma^* = \{0, 1\}$ . Suppose  $R$  and  $S$  are subsets of  $\Sigma^*$  (i.e., they are unary relations.) Further suppose  $R$  and  $S$  are decidable. Prove that  $R \circ S$  is decidable (by giving an algorithm that decides  $R \circ S$  in terms of algorithms that decide  $R$  and decide  $S$ ).

*Let  $M$  and  $N$  be algorithms that decide  $R$  and  $S$ , respectively.*

Input  $w \in \Sigma^*$

Algorithm:

For  $i = 0, 1, 2, \dots, |w|$

*( $|w| :=$  length of  $w$ .)*

Let  $u$  be the initial substring of  $w$  of length  $i$

Let  $v$  " " final " " " " " " "  $|w| - i$

*(So  $w = uv$ ,  $|u| = i$  and  $|v| = |w| - i$ .)*

If  $M(u)$  accepts and  $N(v)$  accepts,  
 accept (and halt)

End-for

Reject.