Math 160B - Winter 2022 - Quiz 2 - Upload by 5:00pm, Friday, January 14

Start Time: **Your name:** Answer Key
Stop Time: **Integrity signature:**

Time limit 15 minutes, not counting download and upload. Please add explanation if over 20 minutes.

1. Let the language L have only the symbol $=$ and no non-logical symbols.
Let $\Gamma = \emptyset$ (the emptyset). Let Π be $\{AtLeast_k : k \geq 2\}$.

(a) Show Γ is not complete, by giving an example of a sentence A such that $\Gamma \not\models A$ and $\Gamma \not\models \neg A$.

(b) Show that Π is $\aleph_0$-categorical.

(c) Prove that Π is complete.

(a) $\forall x \forall y (x = y)$ (or: $AtLeast_2$)

(b) Let $\mathfrak{A} \models \Pi$ and $\mathfrak{B} \models \Pi$ be countable models of Π.
Since $AtLeast_k \in \Pi$ for all k, $\mathfrak{A}$ and $\mathfrak{B}$ are infinite. Therefore $|\mathfrak{A}| = |\mathfrak{B}| = \aleph_0$.
Let $\pi : |\mathfrak{A}| \to |\mathfrak{B}|$ be a bijection.
Since there are no non-logical symbols in L, the conditions for π to be an isomorphism are fulfilled.
Thus, $\pi : \mathfrak{A} \cong \mathfrak{B}$, i.e. $\mathfrak{A}$ and $\mathfrak{B}$ are isomorphic.

(c) Π has no finite models and Π is $\aleph_0$-categorical and the language L is countable. Therefore, by the Łos-Vaught test, the theory $Cn\,\Pi$ is complete.
[Definition: $Cn\,\Pi = \{A : \Pi \models A, A \text{ is a sentence}\}$ is the set of consequences of Π.]