

Start Time:

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Integrity signature:

Time limit 15 minutes, not counting download and upload. Please add explanation if over 17 minutes.

1. Let \mathcal{R}' be the structure $\mathcal{R}' = (\mathbb{R}, 0, 1, +, \cdot, <)$. Let T be the theory $\text{Th } \mathcal{R}'$. Prove T has a non-archimedean model.

A non-archimedean model is a structure \mathcal{A} that has an object $a \in |\mathcal{A}|$ such that (a) a is positive, and (b) a is greater than any fixed integer $n \in \mathbb{N}$. Note that \mathcal{R}' is archimedean since every positive $x \in \mathbb{R}$ is less than or equal to n where $n \lceil x \rceil$. More precisely, condition (a) means that $\mathcal{A} \models a > 0$; and condition (b) means that, for all $n \in \mathbb{N}$, $\mathcal{A} \models \underline{n} < a$ where \underline{n} is the term $1 + 1 + \cdots + 1$ representing n .