0. (If not done in lecture.) Prove that the successor function is representable in \( \mathbb{R} \) and in \( \mathbb{Q} \).

1. Prove that \( \mathbb{Q} \) proves \( m + n = n + m \) for all \( n, m \). Similarly for \( m \cdot n = n \cdot m \).

More of the theorem that \( \mathbb{Q} \subset \mathbb{R} \):

\[
\begin{align*}
Q1: & \quad m + n = m + n \\
Q4: & \quad x + 0 = x \\
Q5: & \quad x + SY = S(x + y)
\end{align*}
\]

Next, consider the case of \( \text{zero} \).

\[
S(0) + S(0) = S(S(0))
\]

\[
S(0) + S(0) = S(S(0)) \quad Q5
\]

\[
S(0) + S(0) = S(S(0) + 0) \quad Q5
\]

\[
S(0) + S(0) = S(S(0)) \quad Q5
\]

\[
S(0) + S(0) = S(S(0)) \quad Q5
\]

\[
S(0) + S(0) = S(S(0)) \quad Q5
\]

General case \( m + n = m + n \).

Keep "shuffling" \( S \)'s from \( n \) to the front.

A formal proof would use induction on \( n \).