Math 160B - Winter 2022 - Class Work - In Lecture, February 22
(No upload today.)

0. (If not done in lecture.) Prove that the successor function is representable in R and in Q.

1. Prove that Q **proves** $\underline{m} + \underline{n} = \underline{n+m}$ **for all** n, m**. Similarly for** $\underline{m} \cdot \underline{m} = \underline{n \cdot m}$

More of the theorem that Q ⊨ R

Q_1 $\underline{m} + \underline{n} = \underline{m+n}$

Q_4 $x + 0 = x$

Q_5 $x + Sy = S(x+y)$

$a = b \to Sa = Sb$
equality axiom of FO

$S(S(0)) + S(S(0))$

$S(S(S(0)) + S(0))$

$\vdots$

$S(S(S(S(0))))$

$SS0 + SS0 = SSSS0$ $\quad \underline{2} + \underline{2} = \underline{4}$.

$S(SS0 + S0) = SSSS0$

$SS0 + SS0 = S(SS0 + S0)$ Q_5

$= SS(SS0 + 0)$ Q_1

same term $\longrightarrow$ $= SS(SS0)$ Q_4

which is $SSSS0$
i.e. $S(S(S(S(0))))$

General case $\underline{m} + \underline{n} = \underline{m+n}$

Keep "shuffling" S's from n to the front

A formal proof would use induction on n.