1. True or false? If true, give an algorithm. If false, what is the difficulty in constructing an algorithm? Let $R, S, R_i, \ldots$ be subsets of $\Sigma^*$ where $\Sigma^* = \{0, 1\}$ (or subsets of $\mathbb{N}$). Let $f : \Sigma^* \rightarrow \Sigma^*$ (or $f : \mathbb{N} \rightarrow \mathbb{N}$).

(It is OK use either semidecidability or computable enumerability. Typically it easier to use computable enumerability for hypotheses, and to use semidecidability for conclusions.)

(a) If $R$ and $S$ are c.e. (computably enumerable), then $R \cup S$ is c.e.
(b) If $R$ and $S$ are c.e. (computably enumerable), then $R \cap S$ is c.e.
(c) If $R$ and $S$ are c.e. (computably enumerable), then $R \backslash S$ is c.e.
(d) If $f$ is computable and $R$ is decidable, is $\{w : R(f(w))\}$ decidable?
(e) If $f$ is computable and $R$ is c.e., is $\{w : R(f(w))\}$ c.e?
(f) If each $R_i$ is decidable, then $\bigcup_{\{i\}} \times R_i$ is c.e.

If (c) was true. $S$ is c.e. $\iff$ $\overline{S}$ is c.e.

Hence $S$ is c.e. $\Rightarrow$ $S$ is decidable.

For $\{11, 10\}$: $S = \{w : R(f(w))\}$

$f$ is an example of a "many one reduction" from $S$ to $R$.

(d) Yes, run algorithm for $f$, then for $R$.

(e) Yes, same idea, use the algorithm that semidecide $R_S$ to get an algorithm that semidecide $S$.

(f) "No" $U_{\{i\}} \times R_i = U_{\{i\}} \{<i,w> : w \in R_i\} = \{<i,w> : w \in R_i\}$

Take any $X \subseteq \mathbb{N}$. Let $R_i = \emptyset$ if $i \notin X$.

$B = \{<i,w> : w \in R_i\} = \{<i,w> : i \in X\}$

- There are uncountably many $X \subseteq \mathbb{N}$
- There are countably many c.e. sets.