1. True or false? If true, give an algorithm. If false, what is the difficulty in constructing an algorithm? Let \( R, S, R_i, \ldots \) be subsets of \( \Sigma^* \) where \( \Sigma^* = \{0, 1\} \) (or subsets of \( \mathbb{N} \)). Let \( f: \Sigma^* \to \Sigma^* \) (or \( f: \mathbb{N} \to \mathbb{N} \)).

(It is OK use either semidecidability or computable enumerability. Typically it easier to use computable enumerability for hypotheses, and to use semidecidability for conclusions.)

(a) If \( R \) and \( S \) are c.e. (computably enumerable), then \( R \cup S \) is c.e.
(b) If \( R \) and \( S \) are c.e. (computably enumerable), then \( R \cap S \) is c.e.
(c) If \( R \) and \( S \) are c.e. (computably enumerable), then \( R \setminus S \) is c.e.
(d) If \( f \) is computable and \( R \) is decidable, is \( \{ w : R(f(w)) \} \) decidable?
(e) If \( f \) is computable and \( R \) is c.e., is \( \{ w : R(f(w)) \} \) c.e?
(f) If each \( R_i \) is decidable, then \( \bigcup_{i \in \mathbb{N}} R_i \) is c.e.

If (c) was true. \( S \) is c.e. \( \Rightarrow \) \( S \) is c.e.

Hence \( S \) is c.e. \( \Rightarrow \) \( S \) is decidable.

For \( \{ x \in \mathbb{N} : S_i = \emptyset \} \)

f is an example of a "many one reduction" from \( S \) to \( R \).

(d) - Yes, run algorithm for \( f \), then for \( R \).

(e) Yes, same idea, use the algorithm that semideclicates \( R_i \) to get an algorithm that semideclicates \( S \).

("No") \( \bigcup_{i \in \mathbb{N}} x \in R_i \) = \( \bigcup_{i \in \mathbb{N}} \{ i, w \} : w \in R_i \) = \( \{ i, w \} : w \in R_i \)

Take any \( X \subseteq \mathbb{N} \). Let \( R_i = \emptyset \) if \( i \notin X \)

\[ \{ (i, w) : w \in R_i \} = \{ (i, w) : i \in X \} \]

- There are uncountably many \( X \subseteq \mathbb{N} \)
- There are countably many c.e. sets.