Math 160B - Winters 2022 - Class Work - In Lecture, January 13 (No upload today.)

Name:

PID:

$R, S \subseteq \Sigma^*$ R, S - set of words

Example $R = \{w \in \Sigma^* : |w| \text{ is even}\}$
$S = \{w \in \Sigma^* : |w| \equiv 0 \bmod 3\}$

1. True or false? If true, give an algorithm. If false, what is the difficulty in constructing an algorithm? Let $R, S, R_i, \ldots$ be subsets of Σ^* where $\Sigma^* = \{0, 1\}$ (or subsets of $\mathbb{N}$). Let $f, g : \Sigma^* \to \Sigma^*$ (or $f, g : \mathbb{N} \to \mathbb{N}$).

Algorithm
Run M_R, if it accepts then accept
Run M_S, if it accepts then accept
Else, reject

(a) If R and S are decidable, then $R \cup S$ is decidable. Yes!

~~(b) If R and S are c.e. (computably enumerable), then $R \cup S$ is c.e.~~

(c) If R and S are decidable, then $R \cap S$ is decidable.

~~(d) If R and S are c.e. (computably enumerable), then $R \cap S$ is c.e.~~

$R \cap S$ is $\overline{\overline{R} \cup \overline{S}}$

(e) If R and S are decidable, then $R \backslash S$ is decidable.

~~(f) If R and S are c.e. (computably enumerable), then $R \backslash S$ is c.e.~~

(g) If each R_i is decidable, then $\bigcup_i (\{i\} \times R_i)$ is decidable.

~~(h) If each R_i is decidable, then $\bigcup_i \{i\} \times R_i$ is c.e.~~

Yes

Input w
Algorithm
Run $M_g(w)$ until it produces (outputs) v.
Run $M_f(v)$ - output the answer

(i) If f and g are computable, is $f \circ g$ computable?
composition

(j) If f is computable and R is decidable, is $\{w : R(f(w)\}$ decidable?

~~(k) If f is computable and R is c.e., is $\{w : R(f(w)\}$ c.e.?~~

$\overline{R} := \Sigma^* \setminus R$

(l) If R is decidable, then the complement is also decidable.

For (g) Problem given $\langle i, w \rangle$: is $w \in R_i$?
Is this decidable $\{\langle i, w \rangle : w \in R_i\}$.

In general no.

Issue Need a uniform algorithm which can take i and run R_i (run the algorithm for R_i)

By uniform : $\langle i, w \rangle \mapsto R_i(w)$'s output