Math 160B - Winter 2022 - Class Work - In Lecture, January 11 (No upload today.)

Name:

PID:

1. True or false? If false, give a counterexample. (T is a theory.)

(a) If A is a formula and $T \models A$, then $A \in T$.

(b) If A is a sentence and $T \models A$, then $A \in T$.

(c) If T is complete, then T is consistent. No $\forall$ sentences, $T \models A$ or $T \models \neg A$.

(d) If T has arbitrarily large finite models, then T has an infinite model. True. "overspill theorem"

(e) If T has an infinite model, then T has arbitrarily large finite models. No Example $\{\text{AtLeast}_k\}_{k \geq 2}$

(f) If T is not complete, then T has models which are not isomorphic. True. Consider $\mathfrak{A} \models T \cup \{A\} \Leftarrow$ $\mathfrak{B} \models T \cup \{\neg A\} \Leftarrow$ where $T \cup \{A\}$ and $T \cup \{\neg A\}$ are consistent

(g) If T has an infinite model, then T has models which are not isomorphic.

Th $\mathbb{N}$

2. Give an example of a theory which is κ-categorical for all infinite κ.

3. Give an example of a theory which has models for all uncountable κ, but does not have a countable model.

4. Give an example of a theory which (a) has models of every finite cardinality, (b) has no countable model, and (c) has models of cardinality κ for every uncountable κ.

5. Give an example of a theory which (a) has models of every infinite cardinality, (b) is not $\aleph_0$-categorical, and (c) is κ-categorical for every uncountable cardinal κ.

6. Describe what must hold for all models of T to be isomorphic. (In this case, we say T is "categorical".)

(Problems 4, 5, 6 may well get left over to be homework problems.)

$\mathfrak{A} \models T$ versus $T = \text{Th}\,\mathfrak{A} \stackrel{\text{def}}{=} \{B : \mathfrak{A} \models B, B \text{ is a sentence}\}$

Note $\text{Th}\,\mathfrak{A}$ is a theory.

$\mathfrak{A} \models T$ means $\text{Th}\,\mathfrak{A} \supseteq T$

$\text{Th}\,\mathfrak{A}$ is a complete theory.

We can

T - complete, but models of T are not all isomorphic.

If T complete and has an infinite model, it has many ~~no~~ many non-isomorphic models (different cardinalities)

Also possible: T is complete but not κ-categorical.