

Math 160B - Winter 2022 - Homework 4
Due Saturday, February 5, 11:00pm
(Hand in by uploading to Gradescope)

You can also find these problems in the PDF text.

1. Prove that every infinite c.e. set R has an infinite decidable subset S .
2. Let R be a set (a unary relation) and f be a unary function. The *preimage* $f^{-1}[R]$ of R under f is the set $\{x : f(x) \in R\}$.
 - (a) Suppose R is decidable and f is computable. Prove that $f^{-1}[R]$ is decidable.
 - (b) Suppose R is c.e. and f is partial computable. Prove that $f^{-1}[R]$ is c.e.
3. Let R be a set (a unary relation) and f be a unary function. The *image* $f[R]$ of R under f is the set $\{f(x) : x \in R\}$. Prove that there a decidable set R and a computable function f such that $f[R]$ is undecidable.
4.
 - (a) Prove that the sets Halt_0 , Halt_1 and $\text{Halt}_{\text{Self}}$ are c.e. You will need to use the assumption that there is a universal algorithm U : be sure to point out where you use the universal algorithm, and explain why this is needed.
 - (b) Conclude that these three sets are not co-c.e.