Math 160A - Fall 2021
Midterm 1 — October 21, 2021 — Duration: ≈ 80 minutes (until 12:20pm)

Name:

PID:

Instructions:

- Check that you have all pages in the midterm (Problems 1-8).
- Write your name and student ID number above.
- Write your initials on the upper right corner of each page. This is in case exams are separated during scanning.
- Get a couple of pages of scratch pages from the instructor; they are handed in loose at the end of the exam. Work on scratch pages will **not** be checked, and does not count towards your grade. **Be sure to transfer enough work from scratch sheets to the answer sheets, so we can see how you arrived at your answers. Otherwise you may not receive credit for your answers.**
- Write your answers on the front sides of the pages; do not write on the back sides. Let us know if you need extra space.

Academic Integrity Guidelines: **You must work this exam on your own. You may not use notes, textbooks, online resources, or resources of any kind.**

PL axioms:

PL1: $A \to B \to A$
PL2: $(A \to B \to C) \to (A \to B) \to (A \to C)$
PL3: $\neg A \to A \to B$
PL4: $(\neg A \to A) \to A$

$A \vee B$ and $A \wedge B$ and $A \leftrightarrow B$ stand for $\neg A \to B$ and $\neg(A \to \neg B)$ and $(A \to B) \wedge (B \to A)$.

Initials:

1. Use the variables

c - "Cows have spots"
h - "Horses have manes"
d - "Dogs are shaggy"
p - "Pigs have wings"
r - "Roosters can fly"

and propositional connectives $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$ to express the sentences (a)-(h) as propositional formulas. When necessary, use parentheses to clarify the meaning of the propositional formulas. (You do not need include all the parentheses required in the formal definition of formulas.)

(a) Roosters can fly only if dogs are shaggy.

(b) Roosters can fly unless dogs are shaggy.

(c) Cows have spots if both horses have manes and pigs can fly.

(d) Either roosters can fly or pigs have wings, but not both.

(e) Neither roosters can fly nor pigs have wings.

(f) At least one of the following three items are true: cows have spots, horses have manes, and dogs are shaggy.

(g) At most one of the following three items are true: cows have spots, horses have manes, and dogs are shaggy.

(g) At most two of the following three items are true: cows have spots, horses have manes, and dogs are shaggy.

(h) All three of the following items are true: cows have spots, horses have manes, and dogs are shaggy.

Initials:

2. For each formula (a)-(e) on the left, which formula or formulas on the right is it tautologically equivalent to? (Write your answers, 1-5, on the lines.)

______(a) $p \oplus q$	1: $p \leftrightarrow q$
______(b) $\neg(p \land q)$	2: $p \leftrightarrow \neg q$
______(c) $p \land q \to p \lor q$	3: $p \leftrightarrow p$
______(d) $p \lor q \to p \land q$	4: $q \leftrightarrow p \leftrightarrow q$
______(e) $(q \to p) \land (\neg p \to q)$	5: None of the above.

3. Let A be the formula $(p \to q) \land (q \to r)$.

(a) Give a CNF formula that is tautologically equivalent to A.

(b) Give a DNF formula that is tautologically equivalent to A.

Initials:

4. Give explicit PL-proofs for the following theorems (from hypotheses), by writing out *every* line (that is, *every* formula) in the PL-proof, including any hypotheses. In each case, state how many lines there are in the PL-proof.

(a) $A \vee B, \neg A \vdash B$.

(b) $A \wedge B, \neg B \vdash A$.

(c) $A \wedge B, \neg A \vdash B$.

Initials:

5. Prove that there exist PL-proofs for the following formulas. Explain the steps clearly; e.g., if you use the Deduction Theorem, Proof-by-Contradiction, Proof-by-Cases, Inconsistency, etc., please state this. — You must justify your steps. Do not use the Completeness Theorem as a justification!

(a) $A \vee B \to \neg B \to A$.

(b) $A \wedge B \to A$.

(c) $A \wedge B \to B$.

6. (This problem continues onto the next page!) Recall that $p|q$ denotes "p NAND q" and that $p{\downarrow}q$ denotes "p NOR q".

(a) We proved that $\{\downarrow\}$ is adequate. Give $\{\downarrow\}$-formulas which are tautologically equivalent to:

1. $\neg p$

2. $p \vee q$

3. $p \wedge q$.

(b) Let $\circledast$ be a new binary connective defined by letting the truth value of $p \circledast q$ be the same as the truth value of $Case(p, p|q, p \to q)$. Is $\{\circledast\}$ adequate? If so, give $\{\circledast\}$-formulas that are tautologically equivalent to $\neg p$ and to $p \vee q$. If not, explain ***why*** it is not adequate.

Problem 6 continued:

(c) Let $\odot$ be a new binary connective defined by letting the truth value of $p \odot q$ be the same as the truth value of $Case(p, p|q, p{\downarrow}q)$. Is $\{\odot\}$ adequate? If so, give $\{\odot\}$-formulas that are tautologically equivalent to $\neg p$ and to $p \vee q$. If not, explain ***why*** it is not adequate.

(d) Let f be a new 3-ary binary connective defined by letting the truth value of $f(p, q, r)$ be the same as the truth value of $Case(p, p \to q, q \to r)$. Is $\{f\}$ adequate? If so, give $\{f\}$-formulas that are tautologically equivalent to $\neg p$ and to $p \vee q$. If not, explain ***why*** it is not adequate.

Initials:

7. State the Compactness Theorem. For full credit, state both forms of the Compactness Theorem. (You may state them either in terms of tautological implication and satisfiability, or in terms of provability and consistency. The Soundness and Completeness Theorems imply that these two ways of expressing the Compactness Theorem are equivalent.)

8. Suppose that Γ and Δ are sets of propositional formulas. Also suppose that $\Gamma \cup \Delta$ is unsatisfiable. Prove that there is a formula A such that $\Gamma \vDash A$ and $\Delta \vDash \neg A$.