Math 160A - Fall 2021 - Quiz 12 - Upload by 9:30am, Wednesday, December 1

Start Time: **Your name:** Answer Key.

Stop Time: **Integrity signature:**

Time limit 15 minutes, not counting download and upload. Please add explanation if over 17 minutes.

FO axioms and inferences:

PL1: $A \to B \to A$

PL2: $(A \to B \to C) \to (A \to B) \to (A \to C)$

PL3: $\neg A \to A \to B$

PL4: $(\neg A \to A) \to A$

EQ1: $x = x$

EQ2: $x = y \to y = x$

EQ3: $x = y \to y = z \to x = z$

EQ$_f$: $y_1 = z_1 \to \cdots \to y_k = z_k \to f(y_1, \ldots, y_k) = f(z_1, \ldots, z_k)$

EQ$_P$: $y_1 = z_1 \to \cdots \to y_k = z_k \to P(y_1, \ldots, y_k) \to P(z_1, \ldots, z_k)$

UI: $\forall x\, A(x) \to A(t)$.

MP: $A \to B,\ A \ /\ B$.

Gen: $C \to A \ /\ C \to \forall x\, A$ (x not free in C)

- $A \vee B$, $A \wedge B$, and $\exists x\, A$ stand for $\neg A \to B$, $\neg(A \to \neg B)$ and $\neg \forall x\, \neg A$.

1. Let P and Q be unary predicate symbols, and 0 be a constant symbol. Let x, y, and y be distinct variables. Give an ***explicit*** FO proof (by listing all the formulas in the proof) of

$$\forall x\, (P(x) \to Q(x)),\ \forall y\, P(y)\ \vdash\ P(0) \to \forall z\, Q(z).$$

Proof from hypotheses $\forall x (P(x) \to Q(x))$, $\forall y P(y)$.

$\forall y P(y)$	Hyp (Hypothesis
$\forall y P(y) \to P(z)$	UI axiom
$P(z)$	Modus Ponens
$\forall x (P(x) \to Q(x))$	Hyp.
$\forall x (P(x) \to Q(x)) \to P(z) \to Q(z)$	UI axiom
$P(z) \to Q(z)$	Modus Ponens
$Q(z)$	Modur Pones
$Q(z) \to P(0) \to Q(z)$	PL1
$P(0) \to Q(z)$	Modus Ponens
$P(0) \to \forall z Q(z)$	Generalization.