

Start Time:

Your name:

Stop Time:

Integrity signature:

Time limit 15 minutes, not counting download and upload. Please add explanation if over 17 minutes.

FO axioms and inferences:

PL1: $A \rightarrow B \rightarrow A$

PL2: $(A \rightarrow B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow (A \rightarrow C)$

PL3: $\neg A \rightarrow A \rightarrow B$

PL4: $(\neg A \rightarrow A) \rightarrow A$

EQ1: $x = x$

EQ2: $x = y \rightarrow y = x$

EQ3: $x = y \rightarrow y = z \rightarrow x = z$

EQ_f: $y_1 = z_1 \rightarrow \dots \rightarrow y_k = z_k \rightarrow f(y_1, \dots, y_k) = f(z_1, \dots, z_k)$

EQ_P: $y_1 = z_1 \rightarrow \dots \rightarrow y_k = z_k \rightarrow P(y_1, \dots, y_k) \rightarrow P(z_1, \dots, z_k)$

UI: $\forall x A(x) \rightarrow A(t)$.

MP: $A \rightarrow B, A / B$.

Gen: $C \rightarrow A / C \rightarrow \forall x A$ (x not free in C)

• $A \vee B, A \wedge B$, and $\exists x A$ stand for $\neg A \rightarrow B, \neg(A \rightarrow \neg B)$ and $\neg \forall x \neg A$.

1. Let P and Q be unary predicate symbols, and 0 be a constant symbol. Let x , y , and z be distinct variables. Give an *explicit* FO proof (by listing all the formulas in the proof) of

$$\forall x (P(x) \rightarrow Q(x)), \forall y P(y) \vdash P(0) \rightarrow \forall z Q(z).$$