

Start Time: Your name: *Answer Key*
 Stop Time: Integrity signature:

Time limit 15 minutes, not counting download and upload. Please add explanation if over 17 minutes.

1. Carry out the following substitutions. As your answer, give the term or formula that results from the substitution. u, v, w, x, y, z are distinct variables.

(a) Let t be the term $f(g(g(0, z), y + z))$. What is $t(0/z)$?

$$f(g(g(0, 0), y + 0))$$

(b) For the same term t , what is $t(f(0), 0/x, z)$?

$$f(g(g(0, 0), y + 0)) \text{ — the same thing as (a).}$$

(c) Let A be $\forall x [y = x \rightarrow \exists y (P(x, y) \rightarrow x = z \vee y = z)]$. What is $A(g(0, u)/z)$?

$$\forall x [y = x \rightarrow \exists y (P(x, y) \rightarrow x = \underline{g(0, u)} \vee y = \underline{g(0, u)})]$$

(d) For the same A , what is $A(f(v), g(0, u)/y, z)$?

$$\forall x [\underline{f(v)} = x \rightarrow \exists y (P(x, y) \rightarrow x = \underline{g(0, u)} \vee y = \underline{g(0, u)})]$$

(e) For the same A , what is $A(g(w, 0), f(v), g(0, u)/x, y, z)$?

Same answer as (d) — x does not appear free in A .

2. Let A be the same formula as above. Which of the following terms are substitutable for y in A ?

- (a) $f(x)$
- (b) y
- (c) 0
- (d) $g(x, y)$

Only (b) & (c)

3. Give an example of an alphabetic variant B of A such that $x + y + z$ is substitutable for z in B .

$$\forall u [y = u \rightarrow \exists v (P(u, v) \rightarrow u = z \vee v = z)]$$

↑ Note "y" is still here