

Start Time: Your name: *Answer Key*
 Stop Time: Integrity signature:

Time limit 15 minutes, not counting download and upload. Please add explanation if over 17 minutes.

For these two questions, answer by specifying the universe of the structure as a set and the interpretation of each non-logical symbol as a set of tuples. Use correct notation for the interpretations, e.g., $E^{\mathcal{A}}$ and $f^{\mathcal{B}}$. (It is **not** required, but you may wish to draw pictures of the structures if you are taking the quiz by hand.)

1. Let E be a binary predicate symbol.
 Give a structure \mathcal{A} such that $\mathcal{A} \models \forall x \forall y (E(x, y) \leftrightarrow \neg E(y, x))$.

Possible answers

- (A) This formula is actually unsatisfiable.
 The intuition is that if $\mathcal{A} \models \forall x \forall y (E(x, y) \leftrightarrow \neg E(y, x))$, then (taking $y=x$), $\mathcal{A} \models \forall x (E(x, x) \leftrightarrow \neg E(x, x))$ which is impossible.
- (B) The intended question was $\mathcal{A} \models \exists x \exists y (E(x, y) \rightarrow \neg E(y, x))$
 One possible answer: $|\mathcal{A}| = \{0\}$, $E^{\mathcal{A}} = \emptyset$.
- (C) Another reasonable assumption was that the problem should be: $\forall x \forall y (x \neq y \rightarrow (E(x, y) \leftrightarrow \neg E(y, x)))$.
 The same structure \mathcal{A} still works.
 Alternate answer: $|\mathcal{A}| = \{0, 1\}$, $E^{\mathcal{A}} = \{\langle 0, 1 \rangle\}$.

2. Let f be a unary function symbol.
 Give a structure \mathcal{B} such that $\mathcal{B} \models \forall x (x \neq f(x) \wedge x \neq f(f(x)))$.

- (A) Possible answer:
 $|\mathcal{B}| = \{0, 1, 2\}$, $f^{\mathcal{B}} = \{\langle 0, 1 \rangle, \langle 1, 2 \rangle, \langle 2, 0 \rangle\}$
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 graph TD
 0 -- f --> 1
 1 -- f --> 2
 2 -- f --> 0

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- (B) Second possible answer:  
 $|\mathcal{B}| = \mathbb{N}$ .  $f^{\mathcal{B}} = \{\langle i, i+1 \rangle : i \in \mathbb{N}\}$