Math 160A - Fall 2021 - Quiz 8 - Upload by 9:30am, Wednesday, November 3

Start Time: **Your name:** Answer Key

Stop Time: **Integrity signature:**

Time limit 15 minutes, not counting download and upload. Please add explanation if over 17 minutes.

1. Use unary predicates $Dog(x)$ and $Cat(x)$, the binary predicate $Likes(x, y)$, the constant symbol *Spot*, the unary function $Mother(x)$ and the equality sign $=$ to express the following English sentences in first-order logic. $Dog(x)$ means "x is a dog" and $Cat(x)$ means "x is a cat". $Likes(x, y)$ means "x likes y". $Mother(x)$ denotes the mother of x. Variables ranges over the universe of all dogs and cats.

(a) All dogs (including Spot) like Spot. $\forall x\,(Dog(x) \to Likes(x, Spot))$

(b) Some dog (possibly it is Spot) likes Spot. $\exists x\,(Dog(x) \wedge Likes(x, Spot))$

(c) Every dog likes the mother of some cat. $\forall x\,(Dog(x) \to \exists y\,(Cat(y) \wedge Likes(x, Mother(y))))$

(d) There is a cat whose mother likes all dogs. $\exists x\,(Cat(x) \wedge \forall y\,(Dog(y) \to Likes(Mother(x), y)))$

2. let P be a unary predicate, Q a binary predicate, f a unary function, g a binary function, and c a constant symbol. Consider the following expressions:

(a) c
(b) x_3
(c) $g(c, x_3)$
(d) $g(c, x_3) =$
(e) $c = g(f(x_2), x_3)$
(f) $g(f(x_1), c) = g(f(x_1))$
(g) $P(c = g(f(x_2), x_3))$
(h) $f(c) = \neg g(x_1, x_1)$
(i) $Q(c, x_1) = P(c)$
(j) $\forall x_1(P(x_1) \to Q(x_1, x_1))$
(k) $\forall x_1 \in P(x_1)\,(Q(x_1, x_1))$
(l) $\forall x_1(Q(x_1, x_1) \wedge x_1 = c)$ ← typo

(i) Which of these are syntactically correct terms? a, b, c

(ii) Which of these are syntactically correct atomic formulas? e

(iii) Which of these are syntactically correct formulas? (A formula still counts as syntactically correct if some parentheses are omitted or extra parentheses added.)
e, j, l (if typo corrected) ; e, j - as written

3. In the following formula, mark which occurrences of x_1 are free occurrences and which are bound occurrences.

$$\forall x_1(x_1 = f(x_1) \to \exists x_1(c = x_1)) \wedge x_1 = f(x_1) \to c = x_1.$$

Neither free nor bound

bound bound free free