Your name: Answer Key **Start Time:**

Stop Time: Integrity signature:

Time limit 15 minutes, not counting download and upload. Please add explanation if over 17 minutes.

- 1. Use unary predicates Dog(x) and Cat(x), the binary predicate Likes(x,y), the constant symbol Spot, the unary function Mother(x) and the equality sign = to express the following English sentences in first-order logic. Dog(x) means "x is a dog" and Cat(x) means "x is a cat". Likes(x,y) means "x likes y". Mother(x) denotes the mother of x. Variables ranges over the universe of all dogs and cats.
 - (a) All dogs (including Spot) like Spot. $\forall_{x} (\mathcal{D}_{6g}(x) \rightarrow \mathcal{L}; les(x, Spot))$
 -]x (Dug(x) n Likes (x, Spot)) (b) Some dog (possibly it is Spot) likes Spot.
 - (c) Every dog likes the mother of some cat. Vx (Dog(x) → Jy (Catly) ~ Likes(x, Mutherly)))
 - (d) There is a cat whose mother likes all dogs. $\exists_{\mathsf{X}} \left(\mathcal{C}_{\mathsf{a}}^{+}(\mathsf{x}) \land \forall_{\mathsf{Y}} \left(\mathcal{D}_{\mathsf{uq}}(\mathsf{y}) \to \mathcal{L}_{\mathsf{i}} \mathsf{kes} \left(\mathcal{M}_{\mathsf{0}} + \mathsf{her}(\mathsf{x}), \mathsf{y} \right) \right) \right)$
- 2. let P be a unary predicate, Q a binary predicate, f a unary function, g a binary function, and c a constant symbol. Consider the following expressions:
 - (a) c

(b) x_3

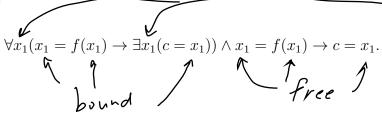
- (e) $c = g(f(x_2), x_3)$ (i) $Q(c, x_1) = P(c)$ (f) $g(f(x_1), c) = g(f(x_1))$ (j) $\forall x_1(P(x_1) \to Q(x_1, x_1))$

(c) $g(c, x_3)$

- (g) $P(c = g(f(x_2), x_3))$
- (k) $\forall x_1 \in P(x_1) (Q(x_1, x_1))$

(d) $g(c, x_3) =$

- (h) $f(c) = \neg g(x_1, x_1)$
- (1) $\forall x_1(Q(x_1, x_1) \land x_1 = c)$
- (i) Which of these are syntactically correct terms?
- (ii) Which of these are syntactically correct atomic formulas?
- (iii) Which of these are syntactically correct formulas? (A formula still counts as syntactically correct if some parentheses are omitted or extra parentheses added. e, j, l (if typo corrected)); le,j-as written
- 3. In the following formula, mark which occurrences of x_1 are free occurrences and which are bound occurrences.



We ther free new bound

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