Start Time: **Your name:**

Stop Time: **Integrity signature:**

Time limit 15 minutes, not counting download and upload. Please add explanation if over 17 minutes.

1. Use unary predicates $Dog(x)$ and $Cat(x)$, the binary predicate $Likes(x, y)$, the constant symbol $Spot$, the unary function $Mother(x)$ and the equality sign $=$ to express the following English sentences in first-order logic. $Dog(x)$ means "x is a dog" and $Cat(x)$ means "x is a cat". $Likes(x, y)$ means "x likes y". $Mother(x)$ denotes the mother of x. Variables ranges over the universe of all dogs and cats.

  (a) All dogs (including Spot) like Spot.

  (b) Some dog (possibly it is Spot) likes Spot.

  (c) Every dog likes the mother of some cat.

  (d) There is a cat whose mother likes all dogs.

2. let P be a unary predicate, Q a binary predicate, f a unary function, g a binary function, and c a constant symbol. Consider the following expressions:

(a) c

(b) x_3

(c) $g(c, x_3)$

(d) $g(c, x_3) =$

(e) $c = g(f(x_2), x_3)$

(f) $g(f(x_1), c) = g(f(x_1))$

(g) $P(c = g(f(x_2), x_3))$

(h) $f(c) = \neg g(x_1, x_1)$

(i) $Q(c, x_1) = P(c)$

(j) $\forall x_1(P(x_1) \to Q(x_1, x_1))$

(k) $\forall x_1 \in P(x_1)\,(Q(x_1, x_1))$

(l) $\forall x_1(Q(x_1, x_1) \land x_1 = c$

  (i) Which of these are syntactically correct terms?

  (ii) Which of these are syntactically correct atomic formulas?

  (iii) Which of these are syntactically correct formulas? (A formula still counts as syntactically correct if some parentheses are omitted or extra parentheses added.)

3. In the following formula, mark which occurrences of x_1 are free occurrences and which are bound occurrences.

$$\forall x_1(x_1 = f(x_1) \to \exists x_1(c = x_1)) \land x_1 = f(x_1) \to c = x_1.$$