

Start Time: Your name: *Answer Key*
 Stop Time: Integrity signature:

Time limit 15 minutes, not counting download and upload. Please add explanation below if over 17 minutes total.

1. Use unary predicates $Dog(x)$ and $Cat(x)$, the binary predicate $Likes(x, y)$, the constant symbol $Spot$, the unary function $Mother(x)$ and the equality sign $=$ to express the following English sentences in first-order logic. $Dog(x)$ means “ x is a dog” and $Cat(x)$ means “ x is a cat”. $Likes(x, y)$ means “ x likes y ”. $Mother(x)$ denotes the mother of x . Variables ranges over the universe of all dogs and cats.

(a) $Spot$ likes every cat. $\forall x (Cat(x) \rightarrow Likes(Spot, x))$

(b) All cats like $Spot$. $\forall x (Cat(x) \rightarrow Likes(x, Spot))$

(c) Some cat likes $Spot$. $\exists x (Cat(x) \wedge Likes(x, Spot))$

(d) At least two cats like $Spot$.
 $\exists x \exists y (x \neq y \wedge Cat(x) \wedge Cat(y) \wedge Likes(x, Spot) \wedge Likes(y, Spot))$

(e) All dogs like each other.
 $\forall x \forall y (x \neq y \wedge Dog(x) \wedge Dog(y) \rightarrow Likes(x, y))$

(f) No dog is the mother of a cat.
 $\neg \exists x \exists y (Dog(x) \wedge Cat(y) \wedge x = Mother(y))$

(g) Each cat that likes themselves also likes $Spot$.
 $\forall x (Cat(x) \wedge Likes(x, x) \rightarrow Likes(x, Spot))$

(h) The mother of any cat is a cat.
 $\forall x (Cat(x) \rightarrow Cat(Mother(x)))$

(i) Every cat and their mother like each other.
 $\forall x (Cat(x) \rightarrow Likes(x, Mother(x)) \wedge Likes(Mother(x), x))$

(j) $Spot$ is not a mother.
 $\forall x (Spot \neq Mother(x))$