Math 160A - Fall 2021 - Quiz #6 - Upload by 9:30am, Wednesday, October 20.

Start Time: **Your name:**

Stop Time: **Integrity signature:**

Time limit 15 minutes, not counting download and upload. Please add explanation below if over 17 minutes total.

PL axioms:
PL1: $A \to B \to A$
PL2: $(A \to B \to C) \to (A \to B) \to (A \to C)$
PL3: $\neg A \to A \to B$
PL4: $(\neg A \to A) \to A$
- $A \lor B$ and $A \land B$ stand for $\neg A \to B$ and $\neg(A \to \neg B)$.

1. Prove that there exists a PL-proof of $A \to B \to A \land B$.

By the Deduction Theorem, twice, it suffices to prove
$A, B \vdash A \land B$.
Expanding the abbreviation $A \land B$, this is the same as
$A, B \vdash \neg(A \to \neg B)$.
By the Principle of Contradiction, it suffices to show
$\Gamma := \{A, B, A \to \neg B\}$ is inconsistent.
Now: $\Gamma \vdash B$ by hypothesis $B \in \Gamma$; and
$\Gamma \vdash \neg B$ by Modus Ponens.
Thus, Γ is inconsistent and the result is proved.

2. Prove that there exists a PL-proof of $\neg(A \land B) \to \neg A \lor \neg B$.

By Deduction Theorem, suffices to prove $\neg(A \land B) \vdash \neg A \lor \neg B$
Expanding the abbreviations, it suffices to show $\neg\neg(A \to \neg B) \vdash \neg\neg A \to \neg B$
By Deduction Theorem, it suffices to show $\neg\neg A, \neg\neg(A \to \neg B) \vdash \neg B$
By Contradiction, suffices to show $\{\neg\neg A, \neg\neg(A \to \neg B), B\}$ is inconsistent
" " " " " $\neg\neg A, B \vdash \neg(A \to \neg B)$
" " " " " $\{\neg\neg A, B, A \to \neg B\}$ is inconsistent
" " " " " $B, A \to \neg B \vdash \neg A$
" " " " " $\{B, A \to \neg B, A\}$ is inconsistent.

This was already proved in problem 1.