Math 160A - Fall 2021 - Quiz #5 - Upload by 9:30am, Friday, October 15.

Start Time: **Your name:** Answer Key
Stop Time: **Integrity signature:**

Time limit 15 minutes, not counting download and upload. Please add explanation below if over 17 minutes total.

PL axioms:
PL1: $A \to B \to A$
PL2: $(A \to B \to C) \to (A \to B) \to (A \to C)$
PL3: $\neg A \to A \to B$
PL4: $(\neg A \to A) \to A$
• $A \vee B$ and $A \wedge B$ stand for $\neg A \to B$ and $\neg(A \to \neg B)$.

An "explicit" proof means showing all the lines in the proof, not just proving that a PL-proof exists. HINT: All three explicit proofs on the quiz should have at most three lines.

1. Give explicit PL-proofs for the following formulas.

(a) $B \to (A \vee B)$. $A \vee B$ is the same as $\neg A \to B$

$B \to (\neg A \to B)$ - PL1 axiom
(one line proof)

(b) $(A \to A) \to (A \to A)$.

$(A \to A \to A) \to (A \to A) \to (A \to A)$ PL2
$(A \to A \to A)$ PL1
$(A \to A) \to (A \to A)$ Modus Ponens

2. Give an explicit PL-proof showing $B \to (A \to A),\ B \vdash A \to A$.

B Hyp
$B \to (A \to A)$ Hyp
$A \to A$ M.P. (Modus Ponens)

3. (With the aid of the deduction theorem.) Prove that $\vdash (A \wedge B \to B) \to A \wedge B \to B$.

By the Deduction Theorem, it suffices to show that

$A \wedge B \to B \vdash A \wedge B \to B$

This has a one-line proof (using the hypothesis).