Math 160A - Fall 2021 - Quiz #4 - Upload by 9:30am, Wednesday, October 13.

Start Time: **Your name:** Answer Key

Stop Time: **Integrity signature:**

Time limit 15 minutes. Please add explanation below if over 17 minutes total.

For all the problems, let Γ be the set of formulas

$$\{p_{i+1} \to p_i : i \geq 1\} \cup \{p_j \to p_k : j \text{ is prime, and } k \text{ is the least prime} > j\}.$$

In other words, Γ is:

- $\{p_2 \to p_1,\, p_3 \to p_2,\, p_4 \to p_3, \ldots\} \cup \{p_2 \to p_3,\, p_3 \to p_5,\, p_5 \to p_7,\, p_7 \to p_{11},\, \ldots\}.$

1. Is Γ satisfiable? If so, describe **all** the truth assignments that satisfy Γ.

Yes, Γ is satisfiable. It has three satisfying assignments.

1. $\varphi(p_i) = T$ for all i:
2. $\varphi(p_i) = F$ for all i.
3. $\varphi(p_1) = T$ and $\varphi(p_i) = F$ for all $i \geq 2$.

2. Does $\Gamma \vDash p_1$?
If so, give the minimal subset Γ_0 of Γ such that $\Gamma_0 \vDash p_1$.

No. (Because the second satisfying assignment above has $\varphi(p_1) = F$.)

3. Does $\Gamma \vDash p_1 \to p_1$?
If so, give the minimal subset Γ_1 of Γ such that $\Gamma_1 \vDash p_1 \to p_1$.

Yes. $p_1 \to p_1$ is a tautology, so $\emptyset \vDash p_1 \to p_1$

$\Gamma_1 = \emptyset$ (the empty set).

4. Does $\Gamma \vDash p_2 \to p_8$?
If so, give the minimal subset Γ_2 of Γ such that $\Gamma_2 \vDash p_2 \to p_8$

Yes.

$\Gamma_2 = \{p_2 \to p_3,\ p_3 \to p_5,\ p_5 \to p_7,\ p_7 \to p_{11},\ p_{11} \to p_{10},\ p_{10} \to p_9,\ p_9 \to p_8\}.$