Math 160A - Fall 2021 - Homework 8
Due Friday, December 3, 11:00pm
(Hand in by uploading to Gradescope)

Your answers may use results proved in class, and results in the (forthcoming) PDF drafts.

1. Suppose that y and t are each substitutable for x in A.

(a) Show that $\forall x\, A \vdash A(t/x)$ by giving an explicit FO-proof of $A(t/x)$ from $\forall x\, A$. (An "explicit" proof means writing out all the formulas in the proof, indicating if they are a hypothesis, if they are an axiom or if they are inferred by Modus Ponens or Generalization.)
(b) Show that $\forall x\, A \vdash \forall y\, A(y/x)$ by giving an explicit proof of $\forall y\, A(y/x)$ from $\forall x\, A$.

2. Prove the following:

(a) $\vdash f(x) = f(x)$.
(b) $\vdash x = y \to u = v \to g(f(x), f(u)) = g(f(y), f(v))$.
(c) $\vdash x = y \to P(y) \to P(x)$.

3. Prove that $\vdash \forall x\, A \to \exists x\, A$.

4. Suppose that $\forall x\, A \to C$ is a sentence. In other words, x does not occur free in C and x is the only variable which appears free in A. Prove:

(a) $\forall x\, (A \to C) \vdash \exists x\, A \to C$.
(b) $\exists x\, A \to C \vdash \forall x\, (A \to C)$.

(This exercise also holds under just the assumption that x is not free in C.)

5. The Deduction Theorem for FO requires A to be a sentence. Show that this hypothesis cannot be eliminated by giving an example of formulas A and B such that $A \vdash B$ but $\nvdash A \to B$. (You can use the Soundness Theorem to prove $\nvdash A \to B$ by showing that $\nvDash A \to B$.)