Math 160A - Fall 2021 - Homework 5 - Due Wednesday, November 3, 10:00pm
(Hand in by uploading to Gradescope)

For problems 2 and 3, L_{PA} is the language $0, S, +, \cdot$.

1. A quote that is sometimes (mis?)attributed to Abraham Lincoln is: "You can fool some of the people all of the time. You can fool all of the people some of the time. But you cannot fool all of the people all of the time." Let these three assertions be:

(a) You can fool some of the people all of the time.
(b) You can fool all of the people some of the time.
(c) You cannot fool all of the people all of the time.

Express these three sentences in first-logic. Let variables range of the universe consisting of all people and all times. Let $Person(x)$ mean x is a person, and $Time(x)$ mean x is a time. Let $F(x, y)$ mean you can fool person x at time y.

Sentences (a) and (b) have two possible translations to first-order logic. Give both translations. Explain how the two translations differ in meaning.

2. Express the following in L_{PA} (over the non-negative integers).

(a) x is less than y.
(b) For any n and m with $m > 0$, there are unique values p and r so that $n = q \cdot m + r$ and $r < m$. (Division algorithm.)

3. Express the following in L_{PA} (over the non-negative integers).

- There are infinitely many primes.

There is no way to talk directly about "infinitely many". Instead, express this by saying there are arbitrarily large primes.

4. Over the language with a unary predicate $P(x)$ and (as always) the equality sign $=$, express the following in first-order logic.

(a) There is a unique x satisfying $P(x)$.
(b) There are at least two objects x that satisfy $P(x)$.
(c) There are at least three objects x that satisfy $P(x)$.
(d) There are exactly two objects x that satisfy $P(x)$.