These are slightly modified versions of Exercises III.14, III.9, III.11, III.12, III.13 and III.21 of draft A.3.c of the PDF text. (The exercise numbers are changing with each draft...)

Your answers may use theorems from the PDF text up through Section III.4, but you may **not** use the Completeness or Soundness Theorems. For the last question, you may use the Compactness Theorem.

**1.** Prove that $\vdash ((p \to q) \to p) \to p$. This tautology is known as Pierce's Law, and is sometimes used as an axiom related to the Law of the Excluded Middle.

**2.** Prove that $\Gamma$ is inconsistent if and only if $\Gamma \vdash \neg(p_1 \to p_1)$.

**3.** Show that the following formulas have PL-proofs.

(a) $(A \wedge B) \to A$.

(b) $(A \wedge B) \to B$.

(c) $A \to B \to A \wedge B$.

**4.** Prove that $\Gamma \cup \{A \wedge B\}$ is inconsistent if and only if $\Gamma \cup \{A, B\}$ is inconsistent. [You may use the results from Exercise 3 in your answer.]

**5.** Prove that $\Gamma \cup \{A \vee B\}$ is inconsistent if and only if $\Gamma \cup \{A\}$ and $\Gamma \cup \{B\}$ are both inconsistent.

**6.** Two sets $\Gamma$ and $\Delta$ of formulas are called "complementary" if every truth assignment $\varphi$, satisfies exactly one of $\Gamma$ and $\Delta$. Suppose $\Gamma$ and $\Delta$ are complementary. Prove there are subsets $\Gamma' \subseteq \Gamma$ and $\Delta' \subseteq \Delta$ such that $\Gamma'$ and $\Delta'$ are finite and such that $\Gamma' \vDash \Gamma$ and $\Delta' \vDash \Delta$. (The notation $\Gamma' \vDash \Gamma$ means $\Gamma' \vDash A$ for all $A \in \Gamma$. You will need to use the Compactness Theorem for this problem.)