Math 160A - Fall 2021 - Homework #1 - Due Wednesday, October 6, 10:00pm
(Hand in by uploading as three problems to Gradescope)

1. (Distributivity or non-distributivity for $\wedge$ over $\leftrightarrow$.) For each of the following either prove it is true, or give a truth assignment φ which shows it is false. (To give a proof that it is true, you may either argue informally, or use truth tables or reduced truth tables.)

(a) $p \wedge (q \leftrightarrow r) \vDash (p \wedge q) \leftrightarrow (p \wedge r)$.

(b) $(p \wedge q) \leftrightarrow (p \wedge r) \vDash p \wedge (q \leftrightarrow r)$.

2. (Semantics of $\leftrightarrow$.)

(a) Show that $\leftrightarrow$ is associative by proving that $p \leftrightarrow (q \leftrightarrow r) \vDash\!\!\dashv (p \leftrightarrow q) \leftrightarrow r$.

(b) Consider a formula
$$p_1 \leftrightarrow p_2 \leftrightarrow p_3 \leftrightarrow \cdots \leftrightarrow p_k.$$

Give a simple characterization of when $\varphi(p_1 \leftrightarrow p_2 \leftrightarrow p_3 \leftrightarrow \cdots \leftrightarrow p_k) = \text{T}$. Your answer should be in terms of the number of i's such that $\varphi(p_i)$ is equal to T or F.

3. (Equivalence to a single formula.) Let Γ be a set of formulas.

(a) Suppose Γ is finite. Prove that there exists a formula A such that $\Gamma \vDash A$ and $A \vDash \Gamma$. (The latter means that for all $B \in \Gamma$, $A \vDash B$.)

(b) Give an example of an infinite, satisfiable Γ for which there does not exist a satisfiable formula A such that $A \vDash \Gamma$.