Math 160A - Mathematical Logic - Winter 2016

Quiz #6 — March 2, 2016

NAME: Answer Key

1. Show that $\forall y E(x,y) \not\models \exists u \exists v \forall y (y = u \vee y = v)$, by giving a structure $\mathfrak{A}$ and (if needed) an object assignment s. Explicitly list the members of $|\mathfrak{A}|$ and $E^{\mathfrak{A}}$. For full credit, choose $|\mathfrak{A}|$ to be as small as possible.

$|\mathfrak{A}| = \{0, 1, 2\}$

$E^{\mathfrak{A}} = \{\langle 0,1\rangle, \langle 0,2\rangle\}$

with $s(x) = 0$.

2. a. Give the definition of "elementary class in the wider sense (EC_Δ)".

Let C be a collection of structures.
C is an EC_Δ iff there is a set T of sentences such that $C = \{\mathfrak{A} : \mathfrak{A} \models T\}$

b. Work in the language containing = (equality) and a unary predicate $P(\cdot)$. Describe a set of sentences illustrating that the class of models $\mathfrak{A}$ in which $P^{\mathfrak{A}}$ is infinite is an elementary class in the wider sense (an EC_Δ).

Let T be:

$$\left\{ \exists x_1 \ldots \exists x_i \left(\bigwedge_{j=1}^{i} P(x_i) \wedge \bigwedge_{j=1}^{i} \bigwedge_{k=1}^{j} x_j \neq x_k \right) \right\}_{i \in \mathbb{N}}$$