Name: ____________________________ ID#:__________________

Math 15A - Discrete Mathematics - Spring 1999

Quiz #7 — May 26

You may NOT use the textbook, notes or other references for this test.

1. (40 pts) For each of the following functions f, state whether f is (a) one-to-one and (b) onto. If it is not one-to-one, give an example of two domain elements where f has the same value. Also, if it is not onto, give an example of an element in the codomain of f, which is not in the range of f. (Watch the domains and codomains carefully!)

   a. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 2x$.
   ANSWER: This is one-to-one and onto.

   b. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$.
   ANSWER: This is neither one-to-one and onto.
   No negative number is in the range of f.
   Also: $f(1) = f(-1)$ (for instance).

   c. $f : \mathbb{N} \to \mathbb{N}$, $f(x) = 2x$.
   ANSWER: This is one-to-one but not onto.
   No odd integer is in the range of f.

   d. Let $g : \mathbb{R}^+ \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = \log_e x$ and $h(x) = e^x$. Then define $f : \mathbb{R}^+ \to \mathbb{R}$ to be $f = h \circ g$.
   ANSWER: f is one-to-one but not onto.
   The range of f is only $\mathbb{R}^+$.

   e. Let $g : \mathbb{R}^+ \to \mathbb{R}$ by $g(x) = \log_e x$ and let $f = g^{-1}$.
   ANSWER: $f : R \to \mathbb{R}^+$ is one-to-one and onto.

2. (20 pts) (a) Let A be a set with n elements and B a set with m elements, with $n \leq m$. How many different one-to-one functions are there with domain A and codomain B?

   ANSWER: $\dfrac{m!}{(m-n)!}$

   (b) Now suppose A has n elements and B has m elements and that $n > m$. How many different one-to-one functions are there with domain A and codomain B?

   ANSWER: Zero. There are no such functions.