Name: ____________________________ ID#:__________________

Math 15A - Discrete Mathematics - Spring 1999

Quiz #6 — May 19 — Answer Key

You may NOT use the textbook, notes or other references for this test.

1. (18 pts) What are the following sets equal to? (you should show the set with all its members explicitly listed). $\mathcal{P}(X)$ denotes the power set of X.

   a. $(\{1,2,3\} \cap \{2,3,4\}) \cup \{1,3\} =$ ANS: $\{1,2,3\}$

   c. $\{1,2,3\} - \{2,3,4\} =$ ANS: $\{1\}$

   d. $\emptyset - \{1,2\} =$ ANS: $\emptyset$

   e. $\{1,2\} \times \{2,3\} =$ ANS: $\{(1,2),(1,3),(2,2),(2,3)\}$

   f. $\emptyset \times \{2,3\} =$ ANS: $\emptyset$

   g. $\mathcal{P}(\{1,2\}) =$ ANS: $\{\emptyset, \{1\}\{2\}\{1,2\}\}$

   h. $\mathcal{P}(\emptyset) =$ ANS: $\{\emptyset\}$

   i. $\mathcal{P}(\mathcal{P}(\emptyset)) =$ ANS: $\{\emptyset, \{\emptyset\}\}$

   j. $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) =$ ANS: $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

2. (16 pts) First: describe the following sets in English, as simply as you can. Second: List 3 elements in each set (or all the elements if it has fewer than 3 members.

   a. $\{n \in \mathbb{Z} : n = 2k \text{ for some } k \in \mathbb{Z}\}$
   ANSWER: The set of even integers.

   b. $\{n \in \mathbb{Z} : n = 2k \text{ for all } k \in \mathbb{Z}\}$
   ANSWER: The empty set

   c. $\{n \in \mathbb{Z} : n \bmod 4 = 0 \text{ or } n \bmod 4 = 2\}$
   ANSWER: The set of even integers.

   d. $\{n \in \mathbb{Z} : n \bmod 4 = 0 \text{ and } n \bmod 4 = 2\}$
   ANSWER: The empty set

3. (4 pts) For each of the following, is it a partition of $\mathbb{Z}$. (Answer "yes" or "no" for each one.)

___No___ a. Is $\mathbb{Z}$ a partition of $\mathbb{Z}$?

___Yes___ b. Is $\{\mathbb{Z}\}$ a partition of $\mathbb{Z}$?

___No___ c. Is $\{\mathbb{Z}^-, \mathbb{Z}^+\}$ a partition of $\mathbb{Z}$?

___Yes___ d. Is $\{\mathbb{Z}^-, \{0\}, \mathbb{Z}^+\}$ a partition of $\mathbb{Z}$?

4. (22pts) State which of the following assertions are true for all sets A, B, C. For the assertions which are false, give a counterexample.

___False___ a. $A \cup (A \cap B) = A \cup B$.

Counterexample: $A = \emptyset$ and $B = \{1\}$.

___False___ b. $A \cup (A \cap B) = A \cap B$.

Counterexample: $A = \{1\}$ and $B = \emptyset$.

___False___ c. $A - (B \cap C) = (A - B) - C$.

Counterexample: $A = \{1\}$ and $B = \{1\}$ and $C = \emptyset$.

___True___ d. $A - (B \cup C) = (A - B) - C$.

___False___ e. If $\emptyset \in A$, then $A = \emptyset$.

Counterexample: $A = \{\emptyset\}$. (Any set A which has $\emptyset$ as a **member** will serve as a counterexample.)