Name: ____________________________ ID#:__________________

Math 15A - Discrete Mathematics - Spring 1999

Quiz #4 — April 28

You may NOT use the textbook, notes or other references for this test. The reverse side of the paper has some excerpts from the "Guidelines on how to write proofs" handout.

1. (10 pts) The notation $x|y$ is used for the predicate "x divides y". Write out (semi-)formally the definition of $x|y$.

   ANSWER: x and y are integers and $\exists$ an integer z such that $y = z \cdot x$.

   NOT-QUITE-AS-GOOD ANSWER: x and y are integers and $y = z \cdot x$ for some integer z.

   REMARK: We only use "x divides y" when x and y are integers.

2. (50 pts) Give a proof of the following theorem, showing all steps of the proof. Your proof may depend on "basic algebra", but should not invoke any of the theorems proved in class on in the textbook.

   Start by re-expressing the theorem in semi-formal language, making all quantifiers explicit, etc.

   Theorem If a, b, c are integers and $a|b$ and $b|c$, then $a|c$.

   REEXPRESSION: $\forall$ integers a, b, c, if $\exists$ an integer n s.t. $b = n \cdot a$ and $\exists$ an integer m s.t. $c = m \cdot b$, then $\exists$ an integer p s.t. $c = p \cdot a$.

   Proof: Let a, b, c be integers and assume that $a|b$ and $b|c$. By the definition of divisibility, there is an integer n such that $b = n \cdot a$ and there is an integer m such that $c = m \cdot a$. Let $p = m \cdot n$. By basic algebra, $c = (n \cdot m)a = p \cdot a$. Since the integers are closed under multiplication, p is an integer. Therefore, $a|c$.
   Q.E.D.

Some general guidelines for writing proofs are:

- Before you try to write a proof, express the theorem in (semi-)formal language, including all the quantifiers, etc., in proper syntax.
- Start the proof with the word "**Proof**". End the proof with "Q.E.D.", abbreviating the Latin for "it is demonstrated".
- Write your proof in correct grammar: use complete sentences, follow correct English grammar, use conventional and syntactically correct mathematical expressions.
- State your assumptions clearly.
- Explain every step that is taken in the proof.
- Don't use the same variable for two different purposes.
- Avoid logical fallacies!