Name: ____________________ ID#: ____________________

Math 15A - Discrete Mathematics - Spring 1999

Quiz #3 — April 21

You may NOT use the textbook, notes or other references for this test. The test continues on the reverse side of the paper.

1. (18 pts) Let $D(x)$ be the predicate "x is a dog", $C(x)$ the predicate "x is a dog with curly hair", $S(x)$ the predicate "x is a dog which sheds a lot", and $P(x)$ the predicate "x is a poodle". Express the following English sentences formally as universal or existential sentences using these predicates, universal and existential quantifiers and negation, conjunction, disjunction and implication signs (but do not use symbols for sets of dogs, just use the predicates listed above):

$\exists x(C(x) \wedge \sim S(x))$ (a) Not all dogs with curly hair shed a lot.

$\forall x(P(x) \to \sim C(x))$ (b) No poodle has curly hair.

$\forall x(P(x) \to C(x))$ (c) Poodles have curly hair.

$\exists x(P(x) \wedge C(x) \wedge S(x))$ (d) Some poodles have curly hair and shed a lot.

$\forall x(C(x) \wedge S(x) \to P(x))$ (e) All curly haired dogs which shed a lot are poodles.

$\forall x(C(x) \wedge \sim S(x) \to P(x))$ (f) For a dog to be a poodle, it is sufficient that it have curly hair and not shed a lot.

2. (27 pts) Rewrite the following statements in (semi-)formal form. Use constructions such as "$\forall$ real numbers x", "$\exists$ positive integer x", "x is divisible by y", "x is prime", "$x \leq y$", etc.

   Sample question: "Every prime integer is odd."
   Possible answer 1: "$\forall$ integers x, if x is prime, then x is odd."
   Possible answer 2: "$\forall$ prime integers x, x is odd."

a. There is a largest integer.

ANSWER: $\exists$ integer x, $\forall$ integers y, $y \leq x$.

b. There is a smallest positive integer.

ANSWER: $\exists$ a positive integer x, $\forall$ positive integers y, $x \leq y$.

c. Every integer is divisible by some prime.

ANSWER: $\forall$ integers x, $\exists$ a prime y, y divides x.

d. There is an integer which is divisible by every integer.

ANSWER: $\exists$ an integer x, $\forall$ integers y, y divides x.

Problem #2 continues on reverse...

Problem #2 continued:

e. No odd integer is a prime.

ANSWER: $\forall$ odd integers x, x is not prime.

f. Being a prime integer is a necessary condition for being odd.

ANSWER: $\forall$ odd integers x, x is prime.

g. Some prime integer is not odd.

ANSWER: $\exists$ a prime integer x, x is not odd.

h. For any real numbers $x < y$, there is a rational number between x and y.

ANSWER: $\forall$ integers $x < y$, $\exists$ rational z, $x < z < y$.

OR: $\forall$ integers x and y, if $x < y$ then $\exists$ rational z, $x < z < y$.

i. Being divisible by all integers is a necessary and sufficient condition for being equal to zero.

ANSWER: $\forall$ integers x, $x = 0$ iff $\forall$ integers y, y divides x.

3. (15 pts) For each argument (i.e., inference) shown below, indicate whether the argument as shown is valid, by writing either "Valid" or "Invalid" on the line next to it.

INVALID

All measurable functions have first derivatives.
No continuous functions are measurable.
$\therefore$ No continuous function has first derivatives.

INVALID

Every continuous function is integrable.
sin is integrable.
$\therefore$ *sin* is continuous.

INVALID

Some continuous function is integrable.
f is a continuous function.
$\therefore$ f is some function and hence integrable.

VALID

Some continuous functions are integrable.
All integrable functions are measurable.
$\therefore$ Some measurable function is continuous.

INVALID

Every integrable function is continuous.
f is continuous.
$\therefore$ f is integrable.