Name: ______________________________ ID#:____________________

Math 15A - Discrete Mathematics - Spring 1999

Quiz #2 — April 13

You may NOT use the textbook, notes or other references for this test. The test continues on the reverse side of the paper.

1. (12 pts) This problem uses the following sentence symbols:
   f = "The object falls when released"; n = "The object has a negative charge"; v = "The object is in a vacuum"; and p = "The laws of physics still work".

   Express the following English sentences as formal formulas using the symbols $\wedge$, $\vee$, $\sim$, $\rightarrow$, $\leftrightarrow$ and the above sentence symbols:

   $f \rightarrow p$ (a) The laws of physics still work if the object falls when released.

   $p \vee v \rightarrow f$ (b) A sufficient condition for the object to fall when released is that the laws of physics still work and the object is in a vacuum.

   $p \rightarrow f$ (c) That the object falls when released is a necessary condition for the laws of physics to still work.

   $f \rightarrow n \wedge v$ (d) The object falls when released only if it has a negative charge and is in a vacuum.

2. (4 pts) Write a formal formula expressing the contrapositive of the assertion in part **d** of problem 1 above.

   ANSWER: $\sim(n \wedge v) \rightarrow \sim f$

3. (20 pts) Let $P(x)$ be the predicate "x is a parrot", $C(x)$ the predicate "x is a brightly colored bird", and $T(x)$ the predicate "x is a bird which is able to talk". Express the following English sentences formally as universal or existential sentences using these predicates, universal and existential quantifiers and negation, conjunction, disjunction and implication signs (but do not use symbols for sets of birds, just use the predicates listed above):

   $\forall x(P(x) \rightarrow \sim C(x))$ (a) No parrot is brightly colored.

   $\exists x(P(x) \wedge \sim T(x))$ (b) Some parrots are unable to talk.

   $\forall x(C(x) \rightarrow P(x))$ (c) Each brightly colored bird is a parrot.

   $\forall x(P(x) \rightarrow C(x) \wedge T(x))$ (d) A necessary condition for something to be a parrot is that it be a brightly colored bird which is able to talk.

   $\forall x(C(x) \wedge P(x) \rightarrow T(x))$ (e) Any brightly colored parrot is able to talk.

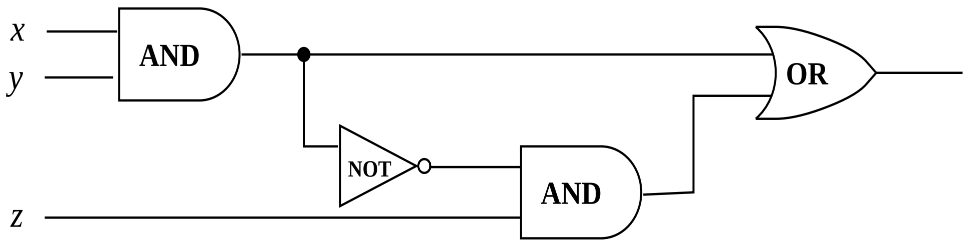
MORE PROBLEMS ON BACK...

4. (8 pts) Write, in formal symbols as in problem 3, the **negations** of the following two sentences:

   a. No parrot is brightly colored. ANSWER: $\exists x(P(x) \wedge C(x))$

   b. Some parrots are unable to talk. ANSWER: $\forall x(P(x) \rightarrow T(x))$

5. (8 pts) Consider the following digital circuit:



   a. Give one set of values for the inputs x, y, z which causes the circuit to output 1 (*True*).

   FIVE POSSIBLE ANSWERS: (1,2) $x = 1,\ y = 1,\ z = 0$ (or $z = 1$)
   (3) $x = 0,\ y = 0,\ z = 1$
   (4) $x = 1,\ y = 0,\ z = 1$
   (5) $x = 0,\ y = 1,\ z = 1$

   b. Write a Boolean formula which is equivalent to the circuit:

   ANSWER: $(x \wedge y) \vee ((\sim(x \wedge y)) \wedge z)$.

6. (8 pts) Draw a digital circuit (built from AND, OR and/or NOT gates) which has three inputs x, y, z and outputs 1 (i.e., *True*) if and only if two or more input values are equal to 1. (Hint: it is possible to do this without using a NOT gate.)

   MORE THAN ONE POSSIBLE ANSWER. HERE IS THE SIMPLEST: