Name: ______________________________ ID#:__________________

Math 15A - Discrete Mathematics - Spring 1999

Final Review Problems

On the final, you may use a *single* sheet of notes (double-sided, $8\frac{1}{2} \times 11$ paper). You may not use the textbook or a calculator. The sheet of notes may not be shared with other students: you must have your own separate sheet of notes.

The final will be cumulative, and cover all the course material, with a little extra emphasis on the material since the midterm. Suggestions for studying: working problems from this sheet, re-working quiz and midterm problems, re-working midterm practice problems, reading the textbook, reviewing assigned homework problems. If you need more to do, you can try working some of the non-assigned problems from the book. The problems on this sheet only supplement the problems from old quizes, the midterm and the homeworks.

1. Let $a_0 = 1$, $a_1 = 1$ and, for $n \geq 2$, $a_n = 2a_{n-2} + a_{n-1}$. Prove that $a_n \leq 2^n$ for all non-negative integers n.

2. For each of the following: either prove or give a counterexample: (U is the universal set)

1. $A \not\subseteq B$ and $B \not\subseteq C$ implies $A \not\subseteq C$.
2. If $A^c \subseteq B$ then $B^c \subseteq A$.
3. If $A^c \subseteq B$ and $B^c \subseteq A$, then $A \cup B = U$.
4. $A - B$ and $B - C$ are disjoint.
5. $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.
6. If R and S are antisymmetric, then $R \cup S$ is antisymmetric.
7. If R and S are antisymmetric, then $R \cap S$ is antisymmetric.

3. Let $f : \{0, 1, 2\} \to \{0, 1, 2, 3\}$ be defined by $f(n) = n + 1$.

1. What is the range of f?
2. Is f one-to-one? Onto?
3. Write f as a set of ordered pairs.

4. Let $\Sigma = \{a, b\}$. Let R be the relation on $A = \Sigma^*$ defined by: uRv iff u is a prefix of v. I.e., uRv iff $v = uw$ for some string $w \in \Sigma^*$. Is R reflexive? Symmetric? Antisymmetric? Transitive? An equivalence relation? A partial order? A total order?

If a total ordering is put on Σ, must the induced lexicographic ordering on Σ^* be compatible with R?

5. Now let A be the set of lines lying in the real plane $\mathbb{R}^2$. And let R be the relation on A defined by: uRv iff u is parallel to v. Is R reflexive? Symmetric? Antisymmetric? Transitive? An equivalence relation? A partial order? A total order?

6. Now let A be the set of positive integers and R be the "divides" relation. Is R reflexive? Symmetric? Antisymmetric? Transitive? An equivalence relation? A partial order? A total order?

Answer the same questions for A the set of non-negative integers (using the convention that $0|0$ is true).

7. Let R be the "divides" relation on $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Draw the Hasse diagram of R. Does R have a greatest element? A least element? A maximal element? A minimal element? If so, list all of these elements.

8. Prove that if $f : A \to B$ is onto and $g : B \to C$ is onto, then $g \circ f$ is onto.

9. Give a one-to-one correspondence between $\mathbb{N}$ and $\mathbb{Z}$. Specify your function as explicitly as you can (with a closed form formula).