Math 155A - Computer Graphics

Homework Assignment, due March 14, 2001

Weighted averages and affine combinations.

- Affine combinations are linear combinations where the coeficients sum to 1.
- Affine combinations are preserved under affine transformations.
- Weighted averages are affine transformations where all the coefficients are $>= 0$.

Study points for barycentric coordinates:

- Barycentric coordinates are *three* values, α, β, γ, and must sum to 1. They are used to linearly interpolate between the vertices of a triangle,

$$\mathbf{u} \ = \ \alpha\mathbf{x}, + \beta\mathbf{y} + \gamma\mathbf{z}.$$

- If two of the barycentric coordinates are equal to zero, then the other coordinate is 1, and $\mathbf{u}$ is equal to the corresponding vertex.
- If one of the the barycentric coordinates is equal to zero, then the other two sum to 1, and the point $\mathbf{u}$ is on the line containing the two points with non-zero coefficient.
- If all three coordinates are > 0, then the point $\mathbf{u}$ is in the interior of the triangle.
- The converses of the last three statements hold too.
- To invert barycentric coordinates, set up two simultaneous linear equations, in β and γ say. Then solve. Don't forget to watch out for simple cases, such as when $\mathbf{u}$ is a vertex or is on an edge.
- Barycentric coordinates can also be used to linearly interpolate a function's values to points $\mathbf{u}$ from the function's values at the three vertices.

Bilinear interpolation.

- Bilinear interpolation gives a method of interpolating between 4 points.
- You are responsible for knowing bilinear interpolation, but not for knowing how to find bilinear interpolation coefficients of interior points of the polygon.

See also the Interpolation Chapter in the online pdf notes for information on the above. It is at

`http://math.ucsd.edu/~sbuss/Math155/notes.html`

Homework problems:

1. Let $\mathbf{x} = \langle 0, -1 \rangle$, $\mathbf{y} = \langle 2, 1 \rangle$, $\mathbf{z} = \langle 0, 1 \rangle$.

a. What point has barycentric coordinates $\alpha = \beta = \gamma = 1/3$?

b. What are the barycentric coordinates of the point $\mathbf{z}$? Of the point $\langle 1, 0 \rangle$? Of the point $\langle 1/2, 1/2 \rangle$?

c. What point has barycentric coordinates $\alpha = \langle -1, 0, 2 \rangle$?

d. Draw a graph showing your answers for parts a.-c. Do your answers appear to be correct?

2. Let $\mathbf{x} = \langle 0, 0 \rangle$, $\mathbf{y} = \langle 1, 0 \rangle$, $\mathbf{z} = \langle 2, 1 \rangle$, $\mathbf{w} = \langle 0, 2 \rangle$.

a. What point is given by bilinear interpolation with $\alpha = 0$ and $\beta = 1/2$?

b. What point is given by bilinear interpolation with $\alpha = 1/3$ and $\beta = 1$?

c. What point is given by bilinear interpolation with $\alpha = 1/3$ and $\beta = 1/3$?

d. Draw the graph showing your answers for parts a.-c. Do your answers appear to be correct?