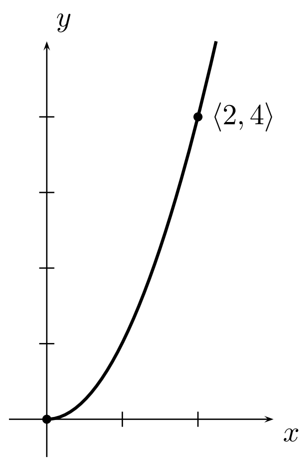
7. [25 points] Consider the knot vector $[0, 0, 0, 1, 1, 2, 2, 2]$ for a degree two B-spline curve.

a. How many control points does this curve use?

b. How many blending functions $N_{i,3}(u)$ does this curve use?

c. For each of these blending functions $N_{i,3}(u)$, state what its support is, and give its formula.

2. [20 points] Consider the parabola $y = x^2$ for $0 \leqslant x \leqslant 2$.



a. Express this portion of the parabola as degree two Bezier curve.

b. Now express this as a degree two B-spline curve.

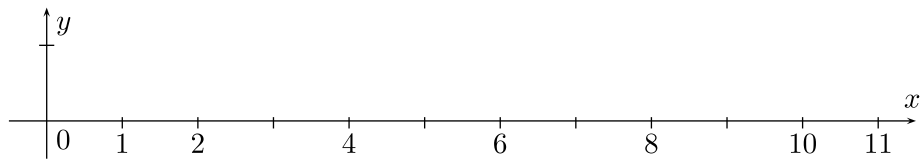
5. [20 points] Consider the knot vector $[0, 0, 0, 1, 1, 1]$.

(a) (Degree zero, order one.) There is one blending function $N_{i,1}$ which is NOT just the constant zero function. Which one is it? (Say which value of i.) Give the formula for this $N_{i,1}$. For this and the questions below, your definition will by "by cases", i.e. depending on the value of i.

(b) (Degree one, order two.) Two of the blending functions $N_{i,2}$ are not just the constant zero function. Which ones are they? Give formulas that define these two functions $N_{i,2}$.

(c) (Degree two, order three.) Three of the blending functions $N_{i,3}$ are not just the constant zero function. Which ones are they? (You do NOT need to give the formulas for these functions. Just say which ones they are.)

1. [20 points] Consider the knot vector 0, 0, 0, 0, 1, 1, 2, 4, 6, 10, 11, 11, 11, 11 for a B-spline curve.



a. If the knot vector is used to define a degree three (order four) B-spline curve, how many control points are needed?

b. What is the domain (support) of the blending function $N_{4,2}(u)$?

c. Draw and label the graph of $N_{4,2}$ on the axes above.

d. Give the formula for $N_{4,2}$ (as a piecewise linear function).

b. What is the domain (support) of the blending function $N_{5,2}(u)$?

e. Draw and label the graph of $N_{5,2}$ on the axes above.

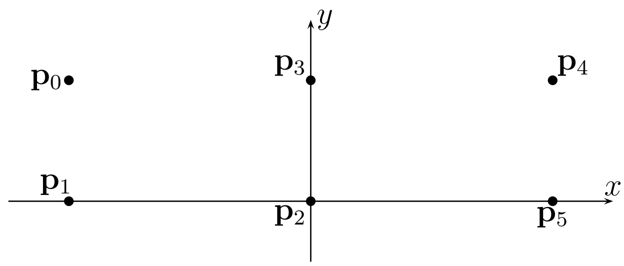
f. What is the domain (support) of the blending function $N_{4,3}(u)$?

g. Draw and label on the axes above a sketch of (the appoximate shape of) the graph of $N_{4,3}(u)$.

3. [20 points] Consider the half-paraboloid $y = x^2 + z^2$ for $x^2 + z^2 \leqslant 4$ and $z \geqslant 0$. This is the *front half* of the surface of rotation formed by rotating the curve from Problem 2 around the y-axis. Express this half-paraboloid as a Bezier patch of degree 2×2 (i.e., order 3×3).

1. [20 points] A cylinder of radius two (2), and height four (4) is placed horizontally centered on the x-axis. The left end of the cylinder is centered at the origin. The right end of the cylinder is centered at $\langle 4, 0, 0 \rangle$. Give a Bezier patch (surface) which defines the front half of the cylinder. You may use any degree Bezier patch you wish.

1. A chord-length parameterization Overhauser curve is defined with the control points $\mathbf{p}_0 = \langle -2, 1\rangle$, $\mathbf{p}_1 = \langle -2, 0\rangle$, $\mathbf{p}_2 = \langle 0, 0\rangle$, $\mathbf{p}_3 = \langle 0, 1\rangle$, $\mathbf{p}_4 = \langle 2, 1\rangle$, and $\mathbf{p}_5 = \langle 2, 0\rangle$.



Main question: What are the control points for the chord-length parameterized Overhauser (sub)curve joining $\mathbf{p}_2$ to $\mathbf{p}_3$? Joining $\mathbf{p}_3$ to $\mathbf{p}_4$?

For partial credit answer the following questions:

a. What is $\mathbf{v}_{3+\frac{1}{2}}$?

b. What is $\mathbf{v}_{2+\frac{1}{2}}$?

c. What is $\mathbf{v}_3$?

d. What is $\mathbf{p}_3^+$?

e. What is $\mathbf{p}_3^-$?

f. By symmetry, what are $\mathbf{p}_2^+$ and $\mathbf{p}_4^-$?

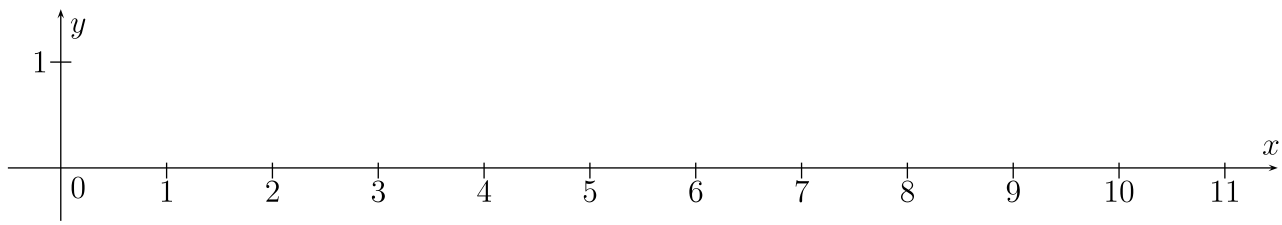
2. Repeat this problem with centripetal parameterization.

1. An ellipsoid $\mathcal{E}$ is equal to $\{\langle x, y, z\rangle : \frac{1}{4}x^2 + y^2 + \frac{1}{4}z^2 = 1$. So $\mathcal{E}$ is centered at the origin, and has radii $2, 1, 2$ along the x-, y-, z-axes, respectively.

Express the front half of $\mathcal{E}$ (where $z \geq 0$) as a rational Bézier patch.

2. For the same ellipsoid $\mathcal{E}$, express the front, right, upper octant of $\mathcal{E}$ as a rational Bézier patch. This is the portion of $\mathcal{E}$ where $x \geq 0$ and $y \geq 0$ and $z \geq 0$.

1. Consider a degree 3 (order 4) B-spline curve $\mathbf{q}(u)$ defined with the knot vector $[0, 1, 2, 3, 4, 5, 5, 6, 7, 8, 9]$. (There is a doubled knot at $u = 5$. So $u_i = i$ for $i \leq 5$ and $u_i = i-1$ for $i > 5$.)



a. What is the domain of $\mathbf{q}(u)$?

b. What is the support of $N_{0,4}(u)$?

c. What is the support of $N_{1,4}(u)$?

d. What is the support of $N_{2,4}(u)$?

e. What is the support of $N_{3,4}(u)$?

f. What is the largest value of i such that $N_{i,4}(u)$ is defined?

g. How many control points are needed for this curve?

h. At $u = 4$, $\mathbf{q}(u)$ must be C^{ℓ}-continuous for (at least) what value ℓ?

i. At $u = 5$, $\mathbf{q}(u)$ must be C^{ℓ}-continuous for (at least) what value ℓ?

j. Draw on the graph above the approximate graph of $N_{1,4}(u)$.

k. Draw on the graph above the approximate graph of $N_{3,4}(u)$.

1. Consider a degree 3 (order 4) B-spline curve $\mathbf{q}(u)$ defined with the knot vector $[0, 0, 0, 0, 1, 2, 2, 3, 3, 3, 4, 5, 5, 5, 5]$.

a. What is the domain of $\mathbf{q}(u)$?

b. What is the domain of $N_{0,4}(u)$?

c. What is the domain of $N_{1,4}(u)$?

d. What is the domain of $N_{2,4}(u)$?

e. What is the domain of $N_{6,4}(u)$?

f. At $u = 1$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for (at least) what value ℓ?

g. At $u = 2$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for (at least) what value ℓ?

h. At $u = 3$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for (at least) what value ℓ?

2. Consider a degree 2 (order 3) B-spline curve $\mathbf{q}(u)$ defined with the knot vector $[0, 0, 0, 1, 2, 2, 3, 3, 3, 4, 5, 5, 5]$. (Similar to above, but with 0 and 5 having only multiplicity three.

a. What is the domain of $\mathbf{q}(u)$?

b. What is the domain of $N_{0,3}(u)$?

c. What is the domain of $N_{1,3}(u)$?

d. What is the domain of $N_{2,3}(u)$?

e. What is the domain of $N_{6,3}(u)$?

f. At $u = 1$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for (at least) what value ℓ?

g. At $u = 2$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for (at least) what value ℓ?

h. At $u = 3$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for (at least) what value ℓ?