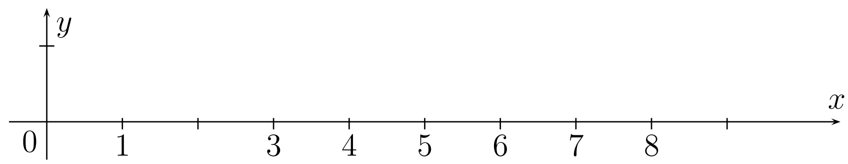
Math 155B — Computer Graphics — Spring 2020
Homework #6 — Due Tuesday, April 28, 9:00pm
Hand in via Gradescope

Note that the multipart question 2 extends onto the second page.

1. Consider the knot vector $[0,0,0,0,1,3,3,4,5,5,5,6,7,8,8,8,8]$ with 17 knots $u_0,\ldots,u_{16}$. E.g., $u_4 = 1$ and $u_7 = 4$ and $u_{11} = 6$. Let this define a **degree three** B-spline curve $\mathbf{q}(u)$ with $n+1$ control points $\mathbf{p}_0,\ldots,\mathbf{p}_n$. Answer questions a.-i. below. You will need to make a copy of this graph:



a. How many control points are needed for this curve? What is the value of n?

b. What is the largest value i such that $N_{i,4}(u)$ is defined?

b. What is the domain of the B-spline curve $\mathbf{q}(u)$?

c. What is the support of $N_{0,4}$?

d. Make a copy of the graph shown above. Draw and label the graph of $N_{0,4}$ on the graph.

e. What is the support of $N_{4,4}$?

f. What is the support of $N_{5,4}$?

g. Draw and label the graph of $N_{5,4}$. You can do this on the same graph as used in part d.

h. For what value ℓ (if any) is it guaranteed that $\mathbf{q}(u)$ is C^ℓ-continuous at $u = 3$?

i. For what pairs of integers i and j must it be that $\mathbf{q}(i) = \mathbf{p}_j$? (Explicitly list all of them.)

2. Consider the knot vector $[0,0,0,0,1,1,1,2,2,2,3,3,3,3]$ for a degree three B-spline curve $\mathbf{q}$.

a. The curve $\mathbf{q}$ uses control points $\mathbf{p}_0,\mathbf{p}_1,\ldots,\mathbf{p}_\ell$. How many control points does this curve use? What is ℓ equal to?

b. How many blending functions $N_{i,4}(u)$ does this curve use?

c. What is the support of $N_{3,4}(u)$? What is the support of $N_{4,4}$? For full credit, correctly indicate whether the endpoints of the intervals of support are included in the supports.

d. The B-spline curve $\mathbf{q}$ can be written as a union of Bézier curves of degree 3. How many Bézier curves are used, and what are their control points?

e. Must the B-spline curve $\mathbf{q}$ be continuous on the domain $[0, 3]$? Must $\mathbf{q}$ be C^1-continuous on the domain $[0, 3]$?

f. The derivative $\mathbf{q}'$ can be expressed as a B-spline curve of degree two. What is the knot vector for $\mathbf{q}'$? Will the derivative $\mathbf{q}'(u)$ be well-defined and continuous for all u in the interval $(0, 3)$? If not, at what values $u \in (0, 3)$ might it not be well-defined and continuous?

3. Page 315 in the textbook (version B.i.k) shows three equations labelled (IX.7) for the blending functions $N_{1,3}(u)$, $N_{2,3}(u)$ and $N_{3,3}(u)$ for the knot vector $0, 0, 0, 0, 1, 1, 1, 1$. Carry out and show the work need to derive these three equations. [You may start with the equations for $N_{2,2}(u)$ and $N_{3,2}(u)$ as given in the text; all of the other functions $N_{i,2}(u)$ (for $i = 0, 1, 4, 5$) are equal to the constant zero function.]

4. Give a full acknowledgement of assistance. This includes anyone, any written source, any web site, etc., that helped you; and anyone you helped.