Math 155B — Computer Graphics — Spring 2020
Homework #5 — Due Wednesday, April 22, 9:00pm
Hand in via Gradescope

1. Let $\mathbf{q}(u)$ be a degree k Bézier curve with control points $\mathbf{p}_0, \ldots, \mathbf{p}_k$; each $\mathbf{p}_i$ is the homogeneous representation of a point $\mathbf{r}_i \in \mathbb{R}^2$. So $\mathbf{p}_i$ has the form $\mathbf{p}_i = \langle x_i, y_i, w_i \rangle$, and then $\mathbf{r}_i = \langle \frac{x_i}{w_i}, \frac{y_i}{w_i} \rangle$.

The value of $\mathbf{q}(u)$ is the form $\langle x(u), y(u), w(u) \rangle$ with $x(u), y(u), w(u)$ degree k polynomials. This is the homogeneous representation a point $\mathbf{r}(u) = \langle \frac{x(u)}{w(u)}, \frac{y(u)}{w(u)} \rangle$ in $\mathbb{R}^2$.

a. Of course $\mathbf{q}'(u) = \langle x'(u), y'(u), w'(u) \rangle$. Give a formula for $\mathbf{r}'(u)$ in terms of the functions $x(u), y(u), w(u)$ and their derivatives.

b. Suppose $\mathbf{p}_0$ is not a point at infinity. Recall that $\mathbf{q}'(0) = k(\mathbf{p}_1 - \mathbf{p}_0)$. Give a formula for $\mathbf{r}'(0)$ in terms of $x_0, y_0, w_0, x_1, y_1, w_1$.

c. (Example.) Suppose for this part that $\mathbf{q}(u)$ is the degree 2 curve with control points $\mathbf{p}_0 = \langle 0, 1, 1 \rangle$, $\mathbf{p}_1 = \langle 1, 0, 0 \rangle$ and $\mathbf{p}_2 = \langle 0, -1, 1 \rangle$, which traces out a half circle. What is $\mathbf{r}'(0)$ equal to?

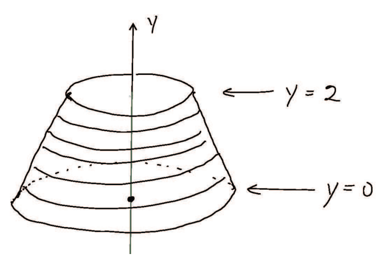
d. Returning to part b., suppose that neither $\mathbf{p}_0$ nor $\mathbf{p}_1$ is a point at infinity. Prove that $\mathbf{r}'(0)$ is a scalar multiple of $k(\mathbf{r}_1 - \mathbf{r}_0)$; namely, $\mathbf{r}'(0) = \alpha k(\mathbf{r}_1 - \mathbf{r}_0)$. What is α equal to?
Note that this means that $\mathbf{r}(u)$ is tangent to the line joining $\mathbf{r}_0$ and $\mathbf{r}_1$ at $u = 0$. (The dual result holds at $u = 1$.)

2. Consider the radius 1 cylinder $\mathcal{C}$ centered on the y-axis. The front half of $\mathcal{C}$ between $y = 0$ and $y = 1$ is the set of points

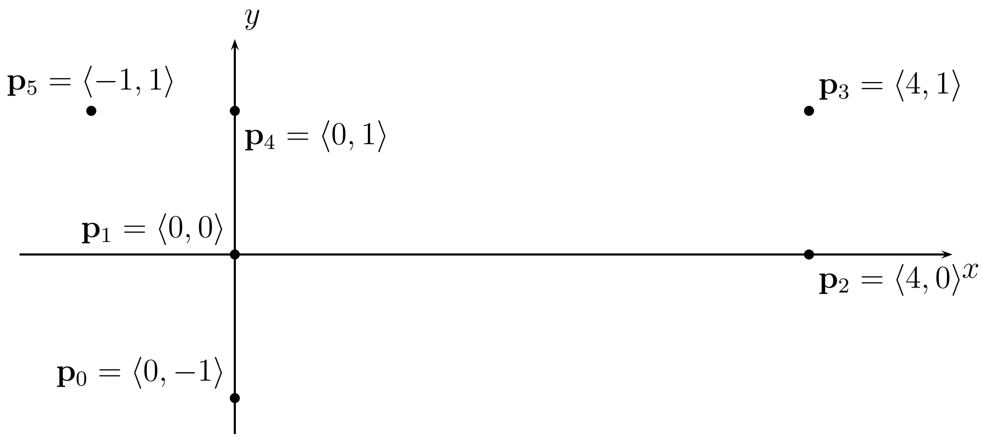
$$\{\langle x, y, z \rangle : x^2 + z^2 = 1 \text{ and } 0 \leq y \leq 1\}.$$

Express this front portion as a Bézier patch. You may use any degree patch you wish, but degree 1×2 is suggested.

3. A truncated cone is centered on the y-axis. Its base is the radius two disk $\{\langle x, 0, z \rangle : x^2 + z^2 \leq 4\}$ lying in the xz plane. Its top is the radius one disk $\{\langle x, 2, z \rangle : x^2 + z^2 \leq 1\}$ lying in the plane $y = 2$. Express the ***front*** of the truncated code as a Bezier patch. You may use any degree Bezier patch you wish.



4. A Catmull-Rom curve is defined with the points $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_5$ as shown on the graph below. (Please see also the next two questions.)



(a) Which of these six points are interpolated by the Catmull-Rom curve?

(b) For each interpolated point $\mathbf{p}_i$, what is the **slope** of the Catmull-Rom curve at that point?

(c) Sketch of the Catmull-Rom curve on the graph above. Be sure to show the slopes at interpolated points clearly.

5. An Overhauser spline curve with **chord-length parameterization** is defined with the same points $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_5$ as in the previous problem.

(a) Which of these six points are interpolated by the Overhauser curve?

(b) For each interpolated point $\mathbf{p}_i$, what is the **slope** of the Overhauser curve at that point?

(c) Sketch of the Overhauser curve on the graph above. Be sure to show the slopes at interpolated points clearly.

6. Now suppose the Overhauser spline uses **centripetal parameterization**. What will the slope of the curve be at $\mathbf{p}_2$?

7. Give a full acknowledgement of assistance. This includes anyone or any web site, etc., that helped you; and anyone you helped.