7. [25 points] Consider the knot vector $[0, 0, 0, 1, 1, 2, 2, 2]$ for a degree two B-spline curve.

a. How many control points does this curve use?

b. How many blending functions $N_{i,3}(u)$ does this curve use?

c. For each of these blending functions $N_{i,3}(u)$, state what its support is, and give its formula.

1. Let $\mathbf{q}(u)$ be a degree 2 Bezier curve with control points $\langle 0, 0, 0\rangle$, $\langle 3, 6, 0\rangle$, and $\langle 6, -6, 0\rangle$. Express $\mathbf{q}(u)$ as a degree three Bezier curve (by giving its control points). Also express $\mathbf{q}(u)$ as a degree four Bezier curve.

2. Suppose a degree 3×2 Bezier patch with control points $\mathbf{r}_{i,j}$ is defined by

$$\mathbf{q}(u) \;=\; \sum_{i=0}^{3} \sum_{j=0}^{2} B_i^3(u) B_j^2(u) \mathbf{r}_{i,j}.$$

Express this patch as a degree 3×3 Bezier patch with control points $p_{i,j}$. Your answer should give formulas for the $\mathbf{p}_{i,j}$'s in terms of the $\mathbf{r}_{i,j}$. Give a brief proof of the correctness of your answer. (Hint: use Theorem VII.8.)

3. Express the "right half" of the unit sphere,

$$\{\langle x, y, z\rangle : x \geq 0 \text{ and } x^2 + y^2 + z^2 = 1\},$$

as a single rational Bezier patch.

Please show work and label answers clearly.

1. Consider a degree 3 (order 4) B-spline curve $\mathbf{q}(u)$ defined with the knot vector $[0, 0, 1, 2, 3, 3, 4, 5, 5, 5, 6, 7, 8, 9, 10]$.

a. How many control points $\mathbf{p}_0, \mathbf{p}_1, \ldots$ are needed to define the curve $\mathbf{q}(u)$?

a. What is the domain of $\mathbf{q}(u)$?

b. What is the domain of $N_{0,4}(u)$?

c. What is the domain of $N_{1,4}(u)$?

d. What is the domain of $N_{2,4}(u)$?

e. What is the domain of $N_{6,4}(u)$?

f. At $u = 2$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for what value ℓ?

g. At $u = 3$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for what value ℓ?

h. At $u = 5$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for what value ℓ?

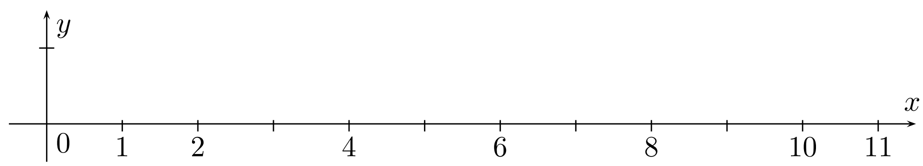
i. At $u = 5.5$, $\mathbf{q}(u)$ must be C^ℓ-continuous, for what value ℓ?

2. [Similar to Exercise VIII.6, page 211. See Figure VIII.9 also on page 211.] Consider a degree 2, order 3 B-spline curve defined with the knot vector $[0, 0, 0, 1, 2, 3, 4, 4, 4]$. How many control points does this curve need to be well-defined? Give the formulas for the following functions:

a. The functions $N_{0,2}$, $N_{1,2}$, $N_{2,2}$, $N_{3,2}$. [Hint: They are piece-wise linear].

b. Only on the interval $[0, 1]$: the functions $N_{0,3}$, $N_{1,3}$, $N_{2,3}$.

2. [28 points] Consider the knot vector $0, 0, 0, 0, 1, 2, 4, 6, 8, 10, 11, 11, 11, 11$ for a B-spline curve.



a. If the knot vector is used to define a degree three (order four) B-spline curve, how many control points are needed?

b. What is the domain (support) of the blending function $N_{4,2}(u)$?

c. Draw and label the graph of $N_{4,2}$ on the axes above.

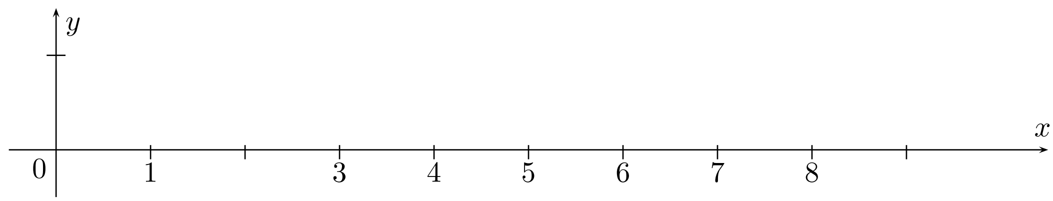
d. Give the formula for $N_{4,2}$ (as a piecewise linear function).

e. Also draw and label the graph of $N_{5,2}$ on the axes above.

f. What is the domain (support) of the blending function $N_{4,3}(u)$?

g. Draw and label on the axes above a sketch of (the appoximate shape of) the graph of $N_{4,3}(u)$.

2. [25 points] Consider the knot vector $[0, 0, 0, 0, 1, 3, 3, 4, 5, 5, 5, 6, 7, 8, 8, 8, 8]$ with 17 knots $u_0, \ldots, u_{16}$. E.g., $u_4 = 1$ and $u_7 = 4$ and $u_{11} = 6$. Let this define a **degree three** B-spline curve $\mathbf{q}(u)$ with $n + 1$ control points $\mathbf{p}_0, \ldots, \mathbf{p}_n$.



a. How many control points are needed for this curve? What is the value of n?

b. What is the largest value i such that $N_{i,4}(u)$ is defined?

b. What is the domain of the B-spline curve $\mathbf{q}(u)$?

c. What is the support of $N_{0,4}$?

d. Draw and label the graph of $N_{0,4}$ on the axes above.

e. What is the support of $N_{4,4}$?

f. What is the support of $N_{5,4}$?

g. Draw and label the graph of $N_{5,4}$ on the axes above.

h. For what value ℓ (if any) is it guaranteed that $\mathbf{q}(u)$ is C^ℓ-continuous at $u = 3$?

i. For what pairs of integers i and j must it be that $\mathbf{q}(i) = \mathbf{p}_j$? (Explicitly list all of them.)

3. [20 points] A degree two Bezier curve $\mathbf{q}(u)$ is defined to trace out the right half of the unit circle in $\mathbb{R}^2$ centered at the origin, with control points $\mathbf{p}_0 = \langle 0, 1, 1 \rangle$, $\mathbf{p}_1 = \langle 1, 0, 0 \rangle$ and $\mathbf{p}_2 = \langle 0, -1, 1 \rangle$.

Each value $\mathbf{q}(u)$ is equal to the homogeneous representation of a point $\mathbf{r}(u)$ in $\mathbb{R}^2$.

What is $\mathbf{r}'(0)$ equal to? (If $\mathbf{q}(u)$ is the homogeneous representation of the position of a point at time u, then $\mathbf{r}'(0)$ is velocity of the point at time 0.)

4. [20 points] Let $\mathbf{q}(u)$ be the same curve as in Problem 3 (right half of the unit circle in $\mathbb{R}^2$).

(a) Express $\mathbf{q}(u)$ as a degree three (order four) Bezier curve by giving its control points.

(b) Express $\mathbf{q}(u)$ as a degree four (order five) Bezier curve by giving its control points.

5. [20 points] Consider the knot vector $[0, 0, 0, 1, 1, 1]$.

(a) (Degree zero, order one.) There is one blending function $N_{i,1}$ which is NOT just the constant zero function. Which one is it? (Say which value of i.) Give the formula for this $N_{i,1}$. For this and the questions below, your definition will by "by cases", i.e. depending on the value of i.

(b) (Degree one, order two.) Two of the blending functions $N_{i,2}$ are not just the constant zero function. Which ones are they? Give formulas that define these two functions $N_{i,2}$.

(c) (Degree two, order three.) Three of the blending functions $N_{i,3}$ are not just the constant zero function. Which ones are they? (You do NOT need to give the formulas for these functions. Just say which ones they are.)

6. [18 points] Answer the questions below either "Yes" or "No" or by selecting the appropriate option i.-iv. The questions assume the shader program has *only* a vertex shader and fragment shader, and also that the shaders does not access any buffers.

a. Can the vertex shader change the position of the vertex?

b. Can the vertex shader determine if the vertex lies in the view volume?

c. When does backface culling occur? (Circle i., ii., or iii.)

i. Before the vertex shader is called.
ii. After the vertex shader is called but before the fragment shader is called.
iii. After the fragment shader is called.

d. When does clipping to the view volume occur? (Circle i., ii., or iii.)

i. Before the vertex shader is called.
ii. After the vertex shader is called but before the fragment shader is called.
iii. After the fragment shader is called.

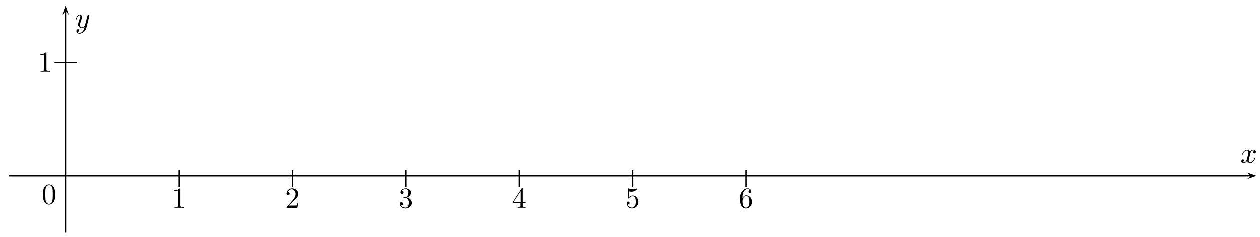
e. When do calls to draw functions such as `glDrawArrays` and `glDrawElements` occur? (Circle i., ii., or iii.)

i. Before the vertex shader is called.
ii. After the vertex shader is called but before the fragment shader is called.
iii. After the fragment shader is called.

f. Gouraud shading (averaging colors for Phong lighting) is done: (Circle i., ii., iii., iv., or v.)

i. Before the vertex shader is called.
ii. By the vertex shader.
iii. After the vertex shader is called but before the fragment shader is called.
iv. By the fragment shader.
v. After the fragment shader is called.

5. [25 points] Consider the knot vector $[0,0,0,0,1,1,1,2,3,3,3,4,5,5,5,5]$ with 16 knots $u_0,\ldots,u_{15}$. E.g., $u_4 = u_6 = 1$ and $u_{11} = 4$. Let this define a **degree three** (order four) B-spline curve $\mathbf{q}(u)$ with $n+1$ control points $\mathbf{p}_0,\ldots,\mathbf{p}_n$.



a. What is the largest value i such that $N_{i,4}(u)$ is defined?

b. What is the domain of the B-spline curve $\mathbf{q}(u)$?

c. What is the support of $N_{0,4}$?

d. Draw and label the graph of $N_{0,4}$ on the axes above – be sure to clearly show its values at the ends of its support.

e. What is the first derivative $N'_{0,4}(0)$ equal to?

f. What is the support of $N_{7,4}$?

g. Draw and label the graph of $N_{7,4}$ on the axes above.

h. For what value ℓ (if any) is it guaranteed that $\mathbf{q}(u)$ is C^{ℓ}-continuous at $u = 3$?

Exercise IX.15. Let $h(x) = 3x^2 + 4x + 5$. What is the degree 2 blossom of h? What is the degree 3 blossom of h? What is the degree 4 blossom of h?

Exercise IX.16. Let $h(x) = 2x^3 + 6x + 1$. What is the degree 3 blossom of h? What is the degree 4 blossom of h?

Exercise IX.18★ Let $h(x) = x^3$. Prove that h does not have a degree 2 blossom.

Exercise IX.17. Let $\mathbf{q}(u)$ be the curve defined on the interval [0, 2] by

$$\mathbf{q}(u) \;=\; \begin{cases} u^2 & \text{if } 0 \le u \le 1 \\ 4u - u^2 - 2 & \text{if } 1 < u \le 2 \end{cases}$$

(a) Verify that $\mathbf{q}(u)$ is C^1-continuous.

(b) We wish to express $\mathbf{q}(u)$ as a degree 2 B-spline curve with knot vector [0, 0, 0, 1, 2, 2, 2]. This requires finding four control points $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$. Use the method of blossoms on the interval [0, 1] to determine $\mathbf{p}_0$, $\mathbf{p}_1$ and $\mathbf{p}_2$.

(c) Use the method of blossoms on the interval $[0, 2]$ to determine $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$. Your answers for $\mathbf{p}_1$ and $\mathbf{p}_2$ should agree with your answers in part (b).

(d) Express the derivative $\mathbf{q}'(u)$ of $\mathbf{q}(u)$ as a degree 1 B-spline curve. What is the domain of $\mathbf{q}'(u)$? What is the knot vector for $\mathbf{q}'(u)$? What is the control points for $\mathbf{q}'(u)$?