Math 155B — Computer Graphics — Spring 2019
Homework #3 — Due 11:00am, Tuesday, June 11
Hand in via GradeScope

1. Consider the rotation matrix $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$. Express this in terms of yaw, pitch and roll in two different ways. Your two answers should not just differ by multiples of 360°.

2. Express the following rotations (expressed as compositions) with yaw (y), pitch (p) and roll (r):

a. $R_{90^\circ,\mathbf{k}} \circ R_{90^\circ,\mathbf{i}} \circ R_{90^\circ,\mathbf{j}}$.

b. $R_{90^\circ,\mathbf{j}} \circ R_{90^\circ,\mathbf{i}} \circ R_{90^\circ,\mathbf{k}}$.

c. $R_{90^\circ,\mathbf{j}} \circ R_{90^\circ,\mathbf{k}} \circ R_{90^\circ,\mathbf{i}}$.

3. Let $q_1 = \langle \frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 0 \rangle$, $q_2 = \langle \frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2} \rangle$, $q_3 = \langle 2, 0, 0, 0 \rangle$, and $q_4 = \langle 1, 0, 1, 0 \rangle$.

a. Calculate $q_1 + q_2$, $q_1 - q_2$, and $q_1 + q_3$.

b. Calculate the products $q_1 q_2$, $q_2 q_1$, $q_1 q_3$ and $q_1(q_2 + q_3)$.

c. Calculate q_1^*, q_2^*, and q_3^*.

d. Calculate $||q_1||$, $||q_2||$, $||q_3||$, $||q_4||$, $||q_1 q_2||$, and $||q_1 q_3||$.

e. Calculate q_1^{-1}, q_2^{-1}, q_3^{-1}, and q_4^{-1}.

4. Let q_1 and q_2 be arbitrary quaternions. Prove that $(q_1 q_2)^* = q_2^* q_1^*$ and that $(q_1 q_2)^{-1} = q_2^{-1} q_1^{-1}$.

5. Let q_1 and q_2 be arbitrary quaternions. Prove that $||q_1 q_2|| = ||q_1|| \cdot ||q_2||$. [Hint: You can give a short proof using the previous exercise and the equality $qq^* = q^* q = ||q||^2$.]

6. Express the matrix from Problem 1 as a unit quaternion in two different ways.