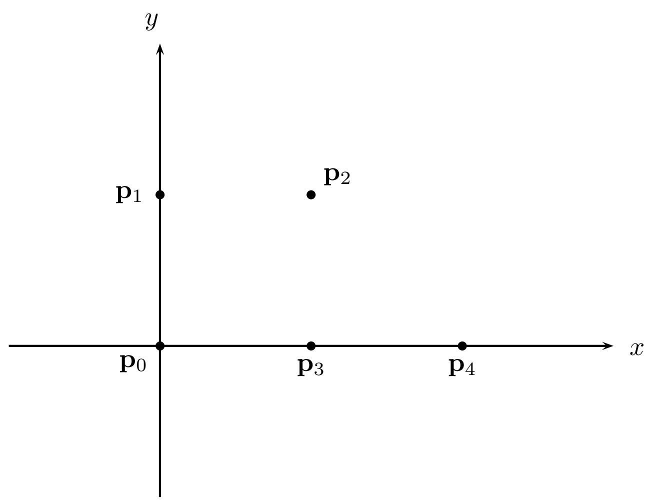
1. Let the Catmull-Rom curve $\mathbf{q}(u)$ be defined by the following control points:

$$\begin{aligned} \mathbf{p}_0 &= \langle 0,0 \rangle \\ \mathbf{p}_1 &= \langle 0,1 \rangle \\ \mathbf{p}_2 &= \langle 1,1 \rangle \\ \mathbf{p}_3 &= \langle 1,0 \rangle \\ \mathbf{p}_4 &= \langle 2,0 \rangle \end{aligned}$$



Thus, $\mathbf{q}(i) = \mathbf{p}_i$ for $i = 1, 2, 3$. For the problems below, show and label all your work, esp. for part d.

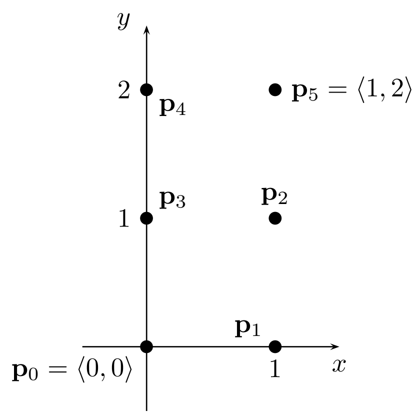
a. Give a freehand sketch of the Catmull Rom curve $\mathbf{q}(u)$ on the graph below.

b. What is the value of $\mathbf{q}'(1)$?

c. What is the value of $\mathbf{q}'(2)$?

d. What is the value of $\mathbf{q}(\frac{3}{2})$?

1. [20 points] A Catmull-Rom curve is defined with the points $\mathbf{p}_0 = \langle 0, 0\rangle$, $\mathbf{p}_1 = \langle 1, 0\rangle$, $\mathbf{p}_2 = \langle 1, 1\rangle$, $\mathbf{p}_3 = \langle 0, 1\rangle$, $\mathbf{p}_4 = \langle 0, 2\rangle$, and $\mathbf{p}_5 = \langle 1, 2\rangle$. This means $\mathbf{q}(i) = \mathbf{p}_i$ for $i = 1, 2, 3, 4$.



a. Draw a freehand sketch of the entire Catmull-Rom curve on the graph. Be sure to show the beginning and ending points and the interpolated points clearly. Also, be sure to indicate the slope correctly at these points.

b. Give the four control points for the Bézier curve which is the portion of the Catmull-Rom curve between the points $\mathbf{p}_1$ and $\mathbf{p}_2$.

c. At the point $\mathbf{p}_2$, is the Catmull-Rom curve C^0-continuous? Is it C^1-continuous there? Is it G^1 continuous there?

5. What methods can be used to determine visibility between patches when computing form factors for radiosity?

3. [5 points] Give the definition of **convex set**.

4. [5 points] Suppose a degree three Bézier curve has control points $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$. Explain what the **convex hull** property tells us about the Bézier curve.

5. [5 points] Suppose a degree three Bézier curve has control points $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$. Explain what the **variation diminishing** property tells us about the Bézier curve.

7. [20 points] In basic recursive ray tracing, a ray intersecting a surface can generate a reflection ray and a transmission ray. Describe, in detail, methods for calculateing the reflection ray and the transmission ray (either by giving algorithms or formulas). Be sure to include the definition of all symbols used.

4. Distributed ray tracing means ray tracing multiple paths. Describe the various uses and applications of distributed ray tracing.

2. [10 points] A piecewise degree three Bézier curve has its first Bézier piece defined with control points $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ and has its second piece defined with control points $\mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$. Of course, $\mathbf{p}_3 = \mathbf{q}_0$.

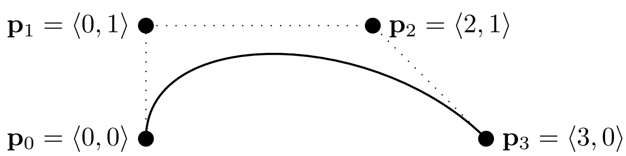
Give necessary and sufficient conditions for the Bézier curve to be C^1-continuous at $\mathbf{p}_3$. Also, give sufficient conditions for the curve to be G^1-continuous at $\mathbf{p}_3$.

1. There are two Bézier patches of degree 3 by 3. The first is defined from the control points $p_{i,j}$, for $0 \leqslant i, j \leqslant 3$. The second is defined from control points $q_{i,j}$, for $0 \leqslant i, j \leqslant 3$. The "top" of the first patch concides with the "bottom" of the second patch, i.e., $p_{i,3} = q_{i,0}$ for $i = 0, 1, 2, 3$.

a. Given a necessary and sufficient condition for the patches to form a surface that is C^1-continuous at their boundary.

b. Give a sufficient condition for the patches to form a surface that is G^1-continuous at their boundary.

2. A degree three Bézier curve has control points and control polygon as pictured. Use recursive subdivision to express the curve as two Bézier curves.



For the next two problems, M is a $n \times n$ matrix, $M = (m_{i,j})$, and B and E are column vectors of length n. The goal is to solve the equation $B = MB + E$ for B.

4. Give the Jacobi algorithm for solving this equation.

5. Give the Gauss-Seidel algorithm for solving this equation.

6. Let q_1 and q_2 be the quaternions $q_1 = 1 + j$ and $q_2 = 2 + 2k$. Compute the following values:

a. $q_1 + q_2$.

b. $q_2 + q_1$.

c. $q_1 q_2$.

d. $q_2 q_1$.

e. q_1^*.

f. q_1^{-1}.

g. $||q_2||$.

7. Let q_1 be as above. What rotation $R_{\theta,\mathbf{u}}$ is represented by the transformation

$$\mathbf{v} \mapsto q_1 \mathbf{v} q_1^{-1}$$

9. Recall that basic ray tracing (in its simplest form) uses only point light sources. On the other hand, radiosity uses only extended light sources, for example, a large rectangular ceiling light illuminating via a uniformly bright frosted cover.

What technique can be used to incorporate this kind of extended light into ray tracing? Explain the technique and give an overview of the algorithm used to implement it.

8. Consider an orientation specified with the parameters:

Yaw is 180 degrees, Pitch is 0 degrees, and Roll is 180 degrees.

a. Express this orientation as a rotation matrix.

b. Express the orientation as a unit quaternion.

9. Recall that basic ray tracing (in its simplest form) uses only point light sources. On the other hand, radiosity uses only extended light sources, for example, a large rectangular ceiling light illuminating via a uniformly bright frosted cover.

What technique can be used to incorporate this kind of extended light into ray tracing? Explain the technique and give an overview of the algorithm used to implement it.

10. A robot arm has two 1-DOF rotational joints as pictured below. Each joint has its rotational axis pointing in the positive z-direction. At a given instant in time, the first joint's position is at $\langle 0,0,0\rangle$ and the second joint's position is at $\langle 2,1,0\rangle$. At the same instant, the end effector's position is at $\langle 2,3,0\rangle$. This configuration is shown in the figure. Thick dark lines are links; dark circles are joints or end effectors.

Either: (a) Give explicitly the Jacobian matrix J for this robot arm at that instant in time. Or: (b) Explain what information you are lacking for this computation.