Math 155A - Computer Graphics - Spring 2022

Midterm #2 — May 26 — Draft version

Name: Answer Key

PID: .....

You must write your initials ON EVERY PAGE. — DO THIS FIRST! This is in case, exams pages get separated during the scanning process. At the same time, please check your copy of the midterm has all XXX problems.

You have 80 minutes. There are XXX problems. You may not use calculators, notes, textbooks, computers, phones, or other resources during this exam.

All answers should be written on the same page as the problem that it answers. Please do not hand in extra pages. Please show enough of your work so that we can see how you obtained your answers.

Use a dark pencil, or a dark blue or black pen so that the test will scan legibly.

Good luck!

1. (Linear Interpolation) Let $\mathbf{x} = \langle -3, -2 \rangle$ and $\mathbf{y} = \langle 3, 7 \rangle$.

a. What is $\textsc{Lerp}(\mathbf{x}, \mathbf{y}, 0)$ equal to? $\langle -3, -2 \rangle$

b. What is $\textsc{Lerp}(\mathbf{x}, \mathbf{y}, \frac{2}{3})$ equal to? $\langle 1, 4 \rangle$

c. What is $\textsc{Lerp}(\mathbf{x}, \mathbf{y}, -1)$ equal to? $\langle -9, -11 \rangle$

d. Let $\mathbf{z} = \langle -1, 1 \rangle$. For what value of α does $\mathbf{z} = \textsc{Lerp}(\mathbf{x}, \mathbf{y}, \alpha)$?

$(1-\alpha)\langle -3, -2 \rangle + \alpha \langle 3, 7 \rangle = \langle -1, 1 \rangle$

$-3 + 3\alpha + 3\alpha = -1$ hence $6\alpha = 2$, so $\alpha = \frac{1}{3}$

or

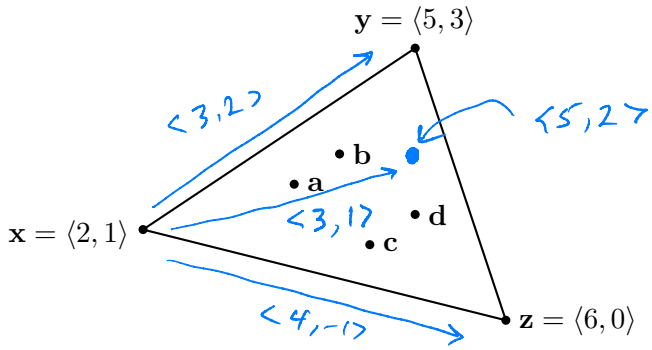
$-2 + 2\alpha + 7\alpha = 1$ hence $9\alpha = 3$, so $\alpha = 1/3$

e. Suppose f is function and it is known that $f(\mathbf{x}) = 2$ and $f(\mathbf{z}) = 12$. If the rest of the values of f are defined by linear interpolation/extrapolation, what is the value of $f(\mathbf{y})$?

We have $\vec{y} = \text{lerp}(\vec{x}, \vec{z}, 3)$ since $z = \text{lerp}(x, y, \frac{1}{3})$.

Therefore $f(\vec{y}) = 32$ is obtained by extrapolation since $\text{lerp}(2, 12, 3) = 32$.

2. Use the following triangle for this question. (It is drawn to scale.) For parts a. and b. give numeric answers. For parts c., d. and e., give the name of a vertex as an answer. For part f., give barycentric coordinates.



a. What point (give xy-coordinates) has barycentric coordinates $\alpha = 1$, $\beta = 0$ and $\gamma = 0$?

$\langle 2, 1\rangle$

b. What point (give xy-coordinates) has barycentric coordinates $\alpha = \frac{1}{2}$, $\beta = \frac{1}{3}$ and $\gamma = \frac{1}{6}$?

$\frac{1}{2}\vec{x} + \frac{1}{3}\vec{y} + \frac{1}{6}\vec{z} = \langle 1 + \frac{5}{3} + 1, \frac{1}{2} + 1 + 0\rangle = \langle \frac{11}{3}, \frac{3}{2}\rangle$

c. Which of the pictured points **a-d** has barycentric coordinates $\alpha = \frac{1}{2}$, $\beta = \frac{1}{3}$ and $\gamma = \frac{1}{6}$?

$\vec{a}$

d. Which of the pictured points **a-d** has barycentric coordinates $\alpha = \frac{1}{6}$, $\beta = \frac{1}{3}$ and $\gamma = \frac{1}{2}$?

$\vec{d}$

e. Which of the pictured points **a-d** has barycentric coordinates $\alpha = \frac{1}{3}$, $\beta = \frac{1}{6}$ and $\gamma = \frac{1}{2}$?

$\vec{c}$

f. What are the barycentric coordinates of $\langle 3, 3\rangle$?

$D =$ Area of $\Delta \vec{x}\vec{y}\vec{z} = \frac{1}{2}\begin{vmatrix} 4 & 3 \\ -1 & 2 \end{vmatrix} = \frac{1}{2} \cdot 11$

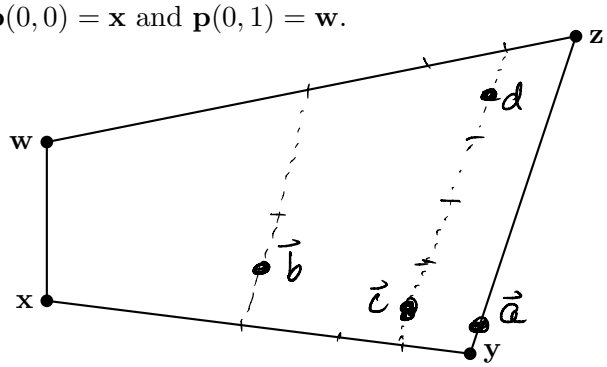
$B =$ Area of $\Delta \vec{x}\langle 5,3\rangle\vec{z} = \frac{1}{2}\begin{vmatrix} 4 & 3 \\ -1 & 1 \end{vmatrix} = \frac{1}{2} \cdot 7$ So $\beta = \frac{7}{11}$

$C =$ Area of $\Delta \vec{x}\vec{y}\langle 5,3\rangle = \frac{1}{2}\begin{vmatrix} 3 & 3 \\ 1 & 2 \end{vmatrix} = \frac{1}{2} \cdot 3$ So $\gamma = \frac{3}{11}$

$\alpha = 1 - \beta - \gamma = \frac{1}{11}$

$$\boxed{\alpha = \frac{1}{11}, \ \beta = \frac{7}{11}, \ \gamma = \frac{3}{11}}$$

3. Bilinear interpolation is used to define a surface $\mathbf{p}(\alpha, \beta)$ from four points $\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}$ in $\mathbb{R}^3$. E.g., $\mathbf{p}(0,0) = \mathbf{x}$ and $\mathbf{p}(0,1) = \mathbf{w}$.



(a) Draw and label on the figure above the approximate locations of the following points $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$. (If necessary, first re-draw the figure on your answer sheet.)

- $\mathbf{a} = \mathbf{p}(1, \frac{1}{10})$,
- $\mathbf{b} = \mathbf{p}(\frac{1}{2}, \frac{1}{4})$,
- $\mathbf{c} = \mathbf{p}(\frac{7}{8}, \frac{1}{8})$,
- $\mathbf{d} = \mathbf{p}(\frac{7}{8}, \frac{7}{8})$.

(b) Now suppose $\mathbf{x} = \langle 1, 0, 1 \rangle$ and $\mathbf{y} = \langle 3, -1, -2 \rangle$ and $\mathbf{z} = \langle 3, 2, 4 \rangle$ and $\mathbf{w} = \langle 1, 2, 1 \rangle$. Give the xyz-components of the point $\mathbf{p}(\frac{1}{3}, \frac{1}{4})$.

$$\frac{3}{4}\left[\frac{2}{3}\vec{x} + \frac{1}{3}\vec{y}\right] + \frac{1}{4}\left[\frac{2}{3}\vec{w} + \frac{1}{3}\vec{z}\right]$$

$$= \frac{3}{4} \cdot \left\langle \frac{5}{3}, \frac{-1}{3}, 0 \right\rangle + \frac{1}{4}\left\langle \frac{5}{3}, 2, 2 \right\rangle$$

$$= \left\langle \frac{5}{3}, \frac{1}{4}, \frac{1}{2} \right\rangle$$

4. For parts (a)-(c), let $\mathbf{x} = \langle 2, 4, 2\rangle$ and $\mathbf{y} = \langle 0, 0, 1\rangle$ be homogeneous representations of $\mathbf{u} = \langle 1, 2\rangle$ and $\mathbf{0}$ in $\mathbb{R}^2$.

(a) What point in $\mathbb{R}^2$ is represented by the homogeneous coordinates $\textsc{Lerp}(\mathbf{x}, \mathbf{y}, \frac{1}{2})$?

$\text{lerp}(\vec{x}, \vec{y}, \frac{1}{2}) = \langle 1, 2, \frac{3}{2}\rangle$. Represents $\boxed{\langle \frac{2}{3}, \frac{4}{3}\rangle}$ in $\mathbb{R}^2$

(b) What point $\mathbf{z}$ in $\mathbb{R}^2$ is $\textsc{Lerp}(\mathbf{u}, \mathbf{0}, \frac{1}{2})$?

$z = \text{lerp}(\vec{u}, \vec{0}, \frac{1}{2}) = \langle \frac{1}{2}, 1\rangle$

(c) For what value of α is $\textsc{Lerp}(\mathbf{x}, \mathbf{y}, \alpha)$ equal to a homogeneous representation of $\mathbf{z}$?

$\boxed{\alpha = \frac{2}{3}}$ - See directly from the weights that $(1-\alpha) = \frac{1}{2}\alpha$ is needed.

Or solve by: $(1-\alpha)\langle 2,4,2\rangle + \alpha\langle 0,0,1\rangle = \langle 2-2\alpha, 4-4\alpha, 2-\alpha\rangle$

Need $\frac{2-2\alpha}{2-\alpha} = \frac{1}{2}$ (so, $4-4\alpha = 2-\alpha$; i.e. $2 = 3\alpha$, so $\alpha = \frac{2}{3}$

Alternately, need $\frac{4-4\alpha}{2-\alpha} = 1$. (So $4-4\alpha = 2-\alpha$, i.e. $\alpha = \frac{2}{3}$).

For part (d), let $\mathbf{a} = \langle a_1, a_2, 2\rangle$ and $\mathbf{b} = \langle b_1, b_2, 6\rangle$. These are homogeneous representations of points $\mathbf{c} = \langle a_1/2, a_2/2\rangle$ and $\mathbf{d} = \langle b_1/6, b_2/6\rangle$ in $\mathbb{R}^2$. Let $\mathbf{m}$ be the midpoint of these points $\mathbf{c}$ and $\mathbf{d}$ in $\mathbb{R}^2$; i.e., $\mathbf{m} = \textsc{Lerp}(\mathbf{c}, \mathbf{d}, \frac{1}{2})$.

(d) For what value of β is $\textsc{Lerp}(\mathbf{a}, \mathbf{b}, \beta)$ equal to a homogeneous representation of $\mathbf{m}$?

Need $2\cdot(1-\beta) = 6\cdot\beta$. Solving for β gives $2 - 2\beta = 6\beta$

so $\boxed{\beta = \frac{1}{4}}$.

5. (Hue; RGB colors) Hue values should be given the range 0 degrees to 360 degrees.

(a) What is the Hue value for the color Green?

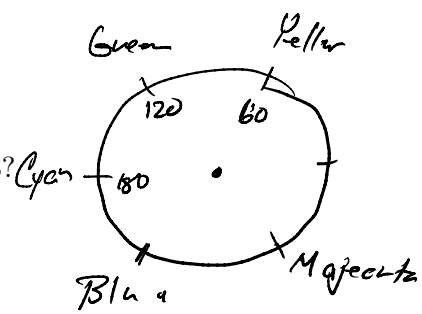
120°

(b) What is the Hue value for the color Magenta?

300°

(c) What is the Hue value for the color with $G = R = 1$ and $B = 0.5$?

60° (Yellow)



(d) What is the Hue value for the color with $R = 0.1$, $G = 0.9$, and $B = 0.3$?

$G = \text{Max}$ (Hence $60 < H < 180$

$R = \text{Min.}$

$H = 120 + \frac{0.2}{0.8} \cdot 60 = \boxed{135^\circ}$

(e) Give an example of a color that has Blue value (B) equal to 0.2, and that has Hue equal to 90°.

$R = 0.6, \quad G = 1.0, \quad B = 0.2$

or $R = 0.3, \quad G = 0.4, \quad B = 0.2$

6. A shearing transformation $\mathbf{f}$ in $\mathbb{R}^2$ is defined by

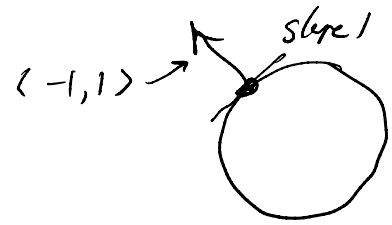
$$\mathbf{f}(\langle x, y\rangle) \;=\; \langle x+y,\, 2y\rangle.$$

(a) Give the 2×2 matrix representing f.

$$\boxed{M = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}} \qquad (M^{-1}) = \frac{1}{2}\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix} \qquad (M^{-1})^T = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$$

(b) Consider the unit circle $\mathcal{C}$ centered at the origin. The function $\mathbf{f}$ transforms C into an ellipse $\mathbf{f}(\mathcal{C})$. The point $\mathbf{u} = \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle$ lies on the unit circle $\mathcal{C}$, and $\mathbf{f}$ maps it to $\mathbf{f}(\mathbf{u}) = \langle 0, \sqrt{2}\rangle$.
The unit circle has slope 1 at the point $\mathbf{u}$.

Give a vector $\mathbf{n}$ which is normal to the ellipse $\mathbf{f}(\mathcal{C})$ at the point $\mathbf{f}(u)$.
What is the slope of the ellipse $\mathbf{f}(\mathcal{C})$ at the point $\mathbf{f}(u)$?



$$\frac{1}{2}\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \frac{1}{2}\cdot\begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

Normal vector is $\langle -1, 1\rangle$

Slope is 1. (Shape has changed but the slope at that point is unchanged!)

7. An ellipsoid $\mathcal{E}$ centered at $\langle 0, 2, 0\rangle$ is defined by the equation

$$2x^2 + (y-2)^2 + 2z^2 = 4.$$

Suppose that $\langle x_0, y_0, z_0\rangle$ lies on the ellipsoid $\mathcal{E}$. Give a formula for a normal vector pointing outward from the surface of the ellipse. Your answer should be in terms of x_0, y_0, z_0. It does not need to be a unit vector.

Let $H(x,y,z) = 2x^2 + (y-2)^2 + 2z^2 - 4$

Gradient $\nabla H = \langle 4x, 2y-4, 4z\rangle$

Answer $\langle 4x_0, 2y_0 - 4, 4z_0\rangle$

8. Consider the following assertions:

a. The halfway vector method works better than the Phong method when the specular exponent is equal to one.

b. Many surfaces act like Lambertian surfaces under diffuse light.

c. Specular light reflectivity is increased for light hitting the surface at a grazing angle.

d. Ambient light is viewed as arriving from all directions.

e. Diffuse and specular light are viewed as coming from a particular direction.

f. The viewer might be very close to the light source, so the ℓ and $\mathbf{v}$ vectors may be nearly equal.

g. Some surfaces are non-Lambertian, such as the moon illuminated by the sun.

h. Phong lighting is a local lighting model and does not model shadows.

i. The ambient and diffuse material properties give the color of a material.

j. Diffuse illumination depends on the light direction (ℓ)view direction ($\mathbf{v}$) but not the view direction ($\mathbf{v}$).

k. Specular highlights depend on the view direction ($\mathbf{v}$) but not the light direction (ℓ).

l. Specular highlights should have the color of the light, not the color of the surface material.

8(a): With **one** exception, the above assertions are true, or true in most circumstances. Which of the above assertions is **false**?

k.

8(b): Which of the above assertions best describes the reason that the Schlick-Fresnel term is sometimes used with Phong lighting:

c.

9. Briefly describe the three listed methods below. Explain how they work. Make it clear how they differ.

(a) Supersampling (non-jittered, non-stochastic).

(b) Stochastic supersampling.

(c) Jittered stochastic supersampling.

See gradescope key.