Math 155A - Computer Graphics - Spring 2022
Midterm #1 — April 28

Name: Answer Key

PID: .....

You must write your initials ON EVERY PAGE. — DO THIS FIRST! This is in case, exams pages get separated during the scanning process. At the same time, please check your copy of the midterm has all eight problems.

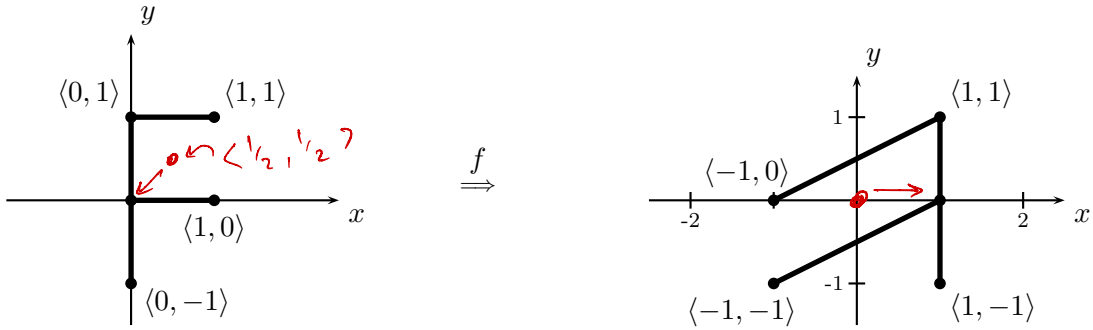
You have 80 minutes. There are 8 problems. You may not use calculators, notes, textbooks, computers, phones, or other resources during this exam.

All answers should be written on the same page as the problem that it answers. Please do not hand in extra pages. Please show enough of your work so that we can see how you obtained your answers.

Use a dark pencil, or a dark blue or black pen so that the test will scan legibly.

Good luck!

1. [10 points] Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be the affine transformation that transforms the standard "F" as shown:



(a) Is f linear? If not, give an example of how it fails to be linear. No. $f(\vec{0}) = \langle 1,0 \rangle \neq 0$ shows it is not linear

(b) Give a 3×3 matrix M that represents f over homogeneous coordinates.

$$\begin{pmatrix} -2 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

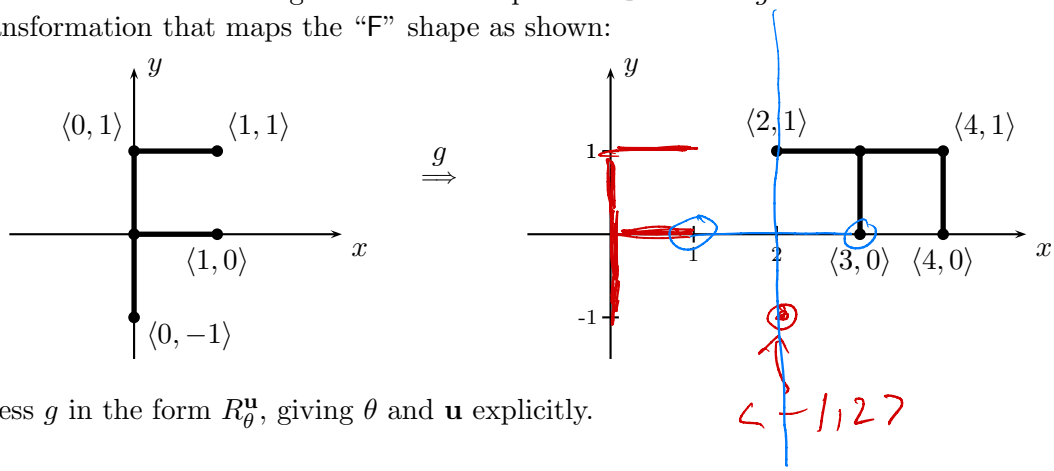
(c) Give a 3×3 matrix M that represents f^{-1} over homogeneous coordinates.

$$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

(d) Is f a rigid transformation? No

(e) Is f an orientation preserving transformation? No

2.[XXX points] Recall that, working in $\mathbb{R}^2$, the generalized rotation transformation $R_\theta^{\mathbf{u}}$ performs a rotation of angle θ around the point $\mathbf{u} \in \mathbb{R}^2$. Let $g : \mathbb{R}^2 \to \mathbb{R}^2$ be the affine transformation that maps the "F" shape as shown:



Express g in the form $R_\theta^{\mathbf{u}}$, giving θ and $\mathbf{u}$ explicitly.

$R^{\langle -1,2\rangle}_{-90^\circ}$

or

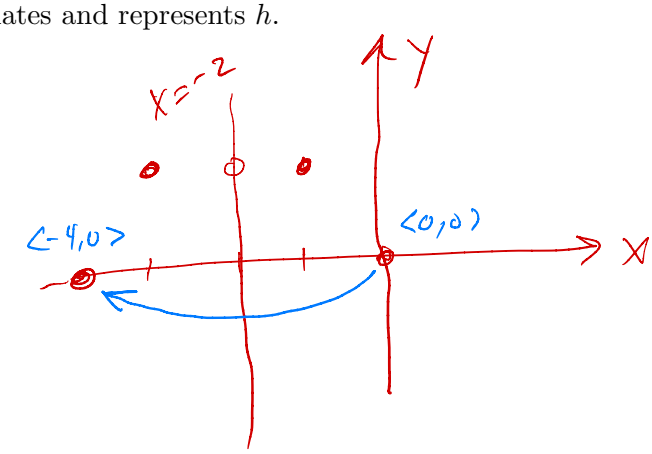
$R^{\langle -1,2\rangle}_{-\pi/2}$

Rotation angle is negative since rotation is clockwise

3. [XXX points] Let $h : \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation that performs a reflection across the plane $P = \{\langle x, y, z\rangle : x = -2\}$.

(a) Give a 4×4 matrix that acts on homogeneous coordinates and represents h.

$\langle x, y, z\rangle \mapsto \langle -4-x, y, z\rangle$



$$\begin{pmatrix} -1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) Express h as a composition of scalings $S_{\langle a,b,c\rangle}$ translations $T_{\mathbf{u}}$, and rotations $R_{\theta,\mathbf{u}}$. Your answer may not need to use all three kinds of transformations. For scalings $S_{\langle a,b,c\rangle}$, it is permitted to have the values a, b, c be negative, positive or zero.

$T_{\langle -4,0,0\rangle} \circ S_{\langle -1,1,1\rangle}$

or $T_{\langle -2,0,0\rangle} \circ S_{\langle -1,1,1\rangle} \circ T_{\langle 2,0,0\rangle}$

or

$S_{\langle -1,1,1\rangle} \circ T_{\langle 4,0,0\rangle}$

Other answers are possible.

4. [XXX points] Let $h : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$h(\langle x, y, z\rangle) \;=\; \left\langle \frac{x+1}{z+1}, 7, \frac{x-y}{z+1} \right\rangle.$$

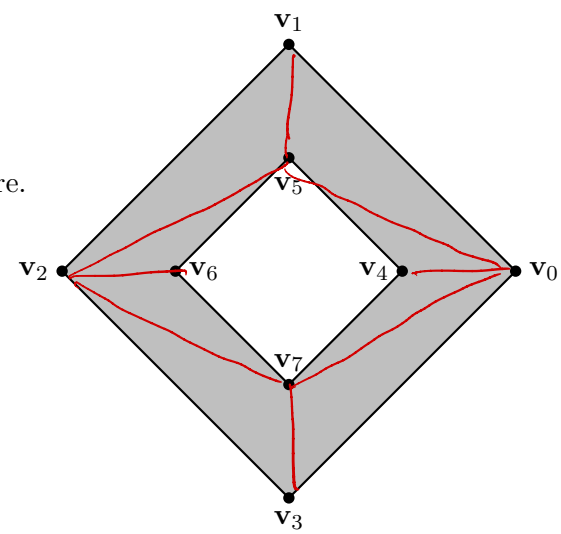
(a) Is h a linear transformation? No

(b) Is h an affine transformation? No

(c) Give a 4×4 matrix that acts on homogeneous coordinates and represents the transformation h.

$$\langle x, y, z, 1\rangle \longmapsto \langle x+1,\ 7z+7,\ x-y,\ z+1\rangle$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 7 & 7 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$



5. [XXX points] Consider the region $\mathcal{R}$ shown in grey in the figure.

(a) Express $\mathcal{R}$ as a triangle strip by listing the vertices used in the triangle strip. The triangle strip should cover the region $\mathcal{R}$ exactly once, and should have front faces pointing towards the viewer.

$V_4\ V_0\ V_5\ V_1\ V_6\ V_2\ V_7\ V_3\ V_4\ V_0$

(b) Express $\mathcal{R}$ as a pair of triangle fans by listing the vertices used in the two triangle fans. The triangle fans should cover points in $\mathcal{R}$ exactly once, and should have front faces pointing towards the viewer.

Centered at V_3 and V_1

$V_3\ V_0\ V_4\ V_7\ V_6\ V_2$ and $V_1\ V_2\ V_6\ V_5\ V_4\ V_0$

or Centered at V_5 and V_7

$V_5\ V_4\ V_0\ V_1\ V_2\ V_6$ and $V_7\ V_6\ V_2\ V_3\ V_0\ V_4$

or etc.

6. [XXX points] Fix $\mathbf{u} \in \mathbb{R}^3$ to be the vector $\mathbf{u} = \langle 2, 1, 0 \rangle$.

Give the 3×3 matrix that represents the linear map in $\mathbb{R}^3$ defined by

$$\mathbf{x} \mapsto \frac{(\mathbf{u} \cdot \mathbf{x})\mathbf{u}}{||\mathbf{u}||^2}$$

$$||u|| = \sqrt{2^2+1^2} = \sqrt{5} \qquad ||u||^2 = 5$$

$$\vec{x} \cdot \vec{u} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = 2x_1 + x_2$$

$$(\vec{x} \cdot \vec{u})\vec{u} = (2x_1 + x_2)\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4x_1 + 2x_2 \\ 2x_1 + x_2 \\ 0 \end{pmatrix}$$

$$\frac{(\vec{x} \cdot \vec{u})\vec{u}}{||u||^2} = \begin{pmatrix} \frac{4}{5}x_1 + \frac{2}{5}x_2 \\ \frac{2}{5}x_1 + \frac{1}{5}x_2 \\ 0 \end{pmatrix}$$

<u>Answer:</u>

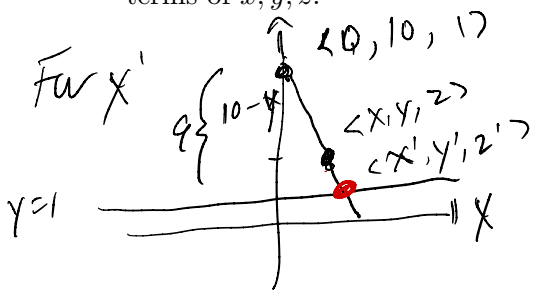
$$\begin{pmatrix} \frac{4}{5} & \frac{2}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

7. [XXX points] A light source is placed at $\langle 0, 10, 1\rangle$ and it casts shadows onto the plane P defined by $y = 1$. This plane is horizontal, parallel to the xz-plane.

When $\langle x, y, z\rangle$ is a point in $\mathbb{R}^3$ with $1 \leqslant y < 10$, define $A(\langle x, y, z\rangle)$ to be the position of the shadow of the point on the plane P. For example, $A(\langle 0, 5, 1\rangle) = \langle 0, 1, 1\rangle$, and $A(\langle 2, 4, 3\rangle) = \langle 3, 1, 4\rangle$.

a. Working in ordinary coordinates (not homogeneous coordinates) give the formula expressing the mapping $A(\langle x, y, z\rangle) = \langle x', y', z'\rangle$. That is, give formulas for x', y', z' in terms of x, y, z.

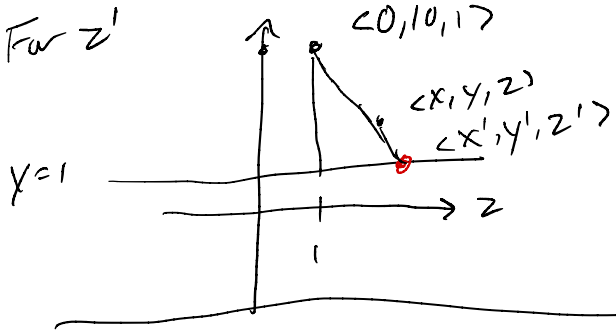
For x'



$$\frac{x'}{9} = \frac{x}{10-y} \qquad x' = \frac{9x}{10-y}$$

$$y' = 1$$

For z'



$$\frac{z'-1}{9} = \frac{z-1}{10-y}$$

$$z' = \frac{9z-9}{10-y} + 1 = \frac{-y+9z+1}{10-y}$$

$$\boxed{A(\langle x,y,z\rangle) = \left\langle \frac{9x}{10-y}, 1, \frac{-y+9z+1}{10-y} \right\rangle}$$

b. Give a 4×4 matrix that represents the transformation A over homogeneous coordinates.

$$\langle x, y, z, 1\rangle \longmapsto \langle 9x, 10-y, -y+9z+1, 10-y\rangle$$

$$\begin{pmatrix} 9 & 0 & 0 & 0 \\ 0 & -1 & 0 & 10 \\ 0 & -1 & 9 & 1 \\ 0 & -1 & 0 & 10 \end{pmatrix}$$

8. [XXX points] (The Painter's Algorithm.)

(a) Briefly explain the Painter's algorithm. What is its purpose? Describe the algorithm. What are its advantages and disadvantages compared with other algorithms for the same purpose?

① Purpose: Hidden surfaces

② Render from back-to-front in order of distance from the viewer.

③ Disadvantages:
- Have to sort triangles
- There may not be a consistent sort order

Advantages
- No need to generically compare all pairs of triangles
- Works well with transparency
- No need for a depth buffer

(b) Describe situations where it might be helpful to use the Painter's algorithm, or methods similar to the Painter's algorithm, in conjunction with the Depth Buffer method.

(1) Transparency works most easily if triangles are rendered in order from most distant to closest

② Depth buffer method can save work in the fragment shaders by rendering in the order of closest to farthest (the reverse of the Painter's algorithm):